HORIZONS IN THE UNIVERSE

I am a part of all that I have met;
Yet all experience is an arch where’ertho’
Gleams that untravelled world, whose margin fades
For ever and for ever when I move.
— Tennyson (1809–92), Ulysses

WHAT ARE COSMIC HORIZONS?

HORIZONS. We look out in space and back in time and do not see the galaxies stretching away endlessly to an infinite distance in the infinite past. We look out to a finite distance and see only those things that are within the observable universe. Like the sea-watching folk in Robert Frost’s poem, we “cannot look out far” and “cannot look in deep.”

The observable universe is usually only a portion of the whole universe. We are at the center of our observable universe, and its distant boundary is a horizon beyond which lie objects that cannot be observed. Observers in other galaxies are located at the centers of their observable universes, which are bounded by horizons. A person on a ship at sea observes the sea stretching away to the horizon, where the sky meets the sea, and is at the center of an “observable sea.” People on other ships at sea are at the centers of their “observable seas,” which are bounded by horizons of a similar nature. Despite this analogy we shall find that cosmic horizons are not quite so simple as the horizon of the sea.

PARTICLE AND EVENT HORIZONS. The subject of cosmic horizons was rather confusing until Wolfgang Rindler cleared up the muddle in 1956. He showed that when we discuss the observable and the unobservable we must realize that there are two kinds of things observed: worldlines and events. Worldlines represent objects such as particles and galaxies that endure over long periods of time; they occupy at any instant a place in space and have extension in time. An event is a brief happening, such as the flash of a firefly or the explosion of a supernova that occupies a place in space and a place in time. Worldlines are in effect strings of events. The events of interest are those that emit rays of light, and the worldlines of interest are those that represent luminous bodies such as galaxies.

To determine what is observable we must specify the nature of the things to be observed. If they are particles or galaxies that endure and therefore have worldlines, we obtain one kind of result; if they are events that occur briefly, we obtain another kind of result. For instance, if one is asked, “Have you met Mr. X?” the answer could be different to that given in response to the question, “Did you see Mr. X at his wedding?” The first question asks if a worldline has been observed, and the second asks if an event has been observed.

There are two types of horizon, a worldline horizon and an event horizon, and both are important. Rindler referred to the worldline horizon as the particle horizon, and as this latter term is now widely adopted we shall also use it. It must be understood,
however, that the word particle in this case means worldline and represents anything that endures. We shall first define the particle and event horizons and then make clear their meaning with the aid of a static universe. Our discussion throughout this chapter is concerned only with horizons in universes that are uniform (i.e., isotropic and homogeneous).

A particle horizon is a spherical surface, with the observer inside at the center, which at a certain instant divides the whole of space into two regions. The region inside contains all galaxies that are observed, and the region outside contains all galaxies that are unobserved. The particle horizon is thus a frontier in space and encloses the observable universe. The horizon at sea is of this type; it is a frontier that divides all ships at sea into two groups: those that are observed and the remainder that are unobserved.

The event horizon divides all events into two groups: those observable at some time or other and those that are never observed. An observer sees events displayed on a backward lightcone. The event horizon is therefore not a surface in space but a backward lightcone that separates events that can be observed from events that are never observed. The event horizon is not quite so obvious as the particle horizon, with its sea-horizon analogy, but this need not cause concern: Subsequent sections will clarify the whole matter.

**HORIZONS IN A STATIC UNIVERSE.** The two types of horizon are most easily demonstrated in an infinite and static universe. Let us forget for the moment that the universe is expanding and suppose that we live in a static Newtonian universe containing uniformly distributed stars. For the sake of simplicity we also suppose that the stars have been luminous for 10 billion years. We can assume either that the universe was created with its luminous stars or that the stars became luminous simultaneously in a preexisting dark universe. The situation is as shown in Figure 19.1. The universe consists of worldlines of luminous stars that commence at the “beginning.”

The worldline labeled $O$ is our star — or Solar System — from which we observe the universe. From $O$, at the instant “now,” we look out in space and back in time and observe other stars in our backward lightcone. We observe the stars because their worldlines intersect our lightcone, and we see each at a particular instant in its lifetime. All stars have been shining for 10 billion years and it is therefore possible to look out and see the stars stretching away to a distance of 10 billion light years. Stars at greater distances cannot be seen because we look back to a time before they existed.

A particle horizon divides all luminous sources into those observed and those unobserved. In the Newtonian universe the particle horizon is therefore at the distance indicated by worldline $X$, and in the present example it lies at a distance of 10 billion light years. Stars at distances less than 10 billion light years are observed and are hence within the observable universe, and stars at greater distances are unobserved and are outside the observable universe.

Let us wait 10 billion years, say — and then we are then at the moment labeled $X$. The particle horizon is $X$, and the observer $O$ can no further than $X$ to see any event. The observable universe, therefore recedes as the observable universe horizon, is expanding.

We turn now to consider whether in the Newtonian universe events that cannot be observed by the observer on a worldline at $X$ exist. In the Newtonian universe the horizon is $X$, and in the present example it lies at a distance of 10 billion light years. Stars at distances less than 10 billion light years are observed and are hence within the observable universe, and stars at greater distances are unobserved and are outside the observable universe.
Figure 19.2. At moment "now" observer O sees no further than worldline X. Subsequently, at the moment labeled "later," the observer sees beyond X to worldline Y. The particle horizon therefore recedes in a static universe, and the observable universe, bounded by the particle horizon, is expanding.

Figure 19.3. A static universe that has an ending. The time labeled "end" is the observer's last possible moment of observation.

If stars shine forever in the future we realize immediately that there is no event horizon. O's lightcone in this case advances up O's worldline, and any point in spacetime that is an event will eventually lie on the lightcone and be observed. Hence, in a static universe in which stars are luminous for an infinite future, there is no event horizon, and every event at some time or other is observed by any observer.

We consider now the case where the Newtonian universe of luminous stars comes to an "end." Either the universe is terminated by its creator, or the stars and observers cease to exist and the universe becomes dark. All worldlines as a result come to an end, as shown in Figure 19.3. It is immediately obvious that in such a universe there is an event horizon, and it is O's lightcone at the last possible moment. Inside the event horizon are the events that can be observed, and outside are the events that are never observed, as shown in Figure 19.4. The lightcone cannot advance further into the future, and all events outside this ultimate lightcone are never seen.

Despite its simplicity, the Newtonian universe serves to illustrate reasonably clearly the nature of cosmic horizons. With it we have learned that beyond the particle
Figure 19.4. The event horizon is the observer’s lightcone when the universe ends. Inside this ultimate lightcone are the events that can be observed, and outside are the events that are never observed.

Figure 19.5. Albert (A) and Bertha (B) have overlapping horizons, but each is able to see things that the other cannot. Can they, by communicating with each other, enlarge their individual horizons into a joint horizon? If they can, then their horizons are not true cosmic horizons.

THE HORIZON RIDDLE

While to deny the existence of an unseen kingdom is bad, to pretend that we know more about it than its bare existence is no better.
— Samuel Butler (1835–1902), Erewhon

Consider two observers, A (for Albert) and B (for Bertha), who are widely separated from each other in the universe. Each has a horizon of some kind such that A cannot see objects beyond his horizon and B cannot see objects beyond her horizon. Although they see each other, each sees objects that the other cannot, as illustrated in Figure 19.5. We now ask: Can B communicate to A information that will extend A’s knowledge of objects beyond his horizon? If so, then surely a third observer C may communicate to B information that B in turn relays to A. A sequence of observers B, C, D, E, . . . may in this way extend A’s knowledge of the universe to indefinite limits. According to this argument A has no horizon! This is the horizon riddle that puzzles many students.

The riddle, it must be admitted, stems from our experience with horizons on the surface of the Earth. Thus if A and B are on ships at sea, within sight of each other, they each observe the sea stretching away to the horizon. A sees objects that B cannot see, and similarly B sees objects that A cannot see; they can communicate with flags and thereby keep each other aware of objects not directly visible. By communication, A and B pool their observations and succeed in extending their horizons. In this way a pre-twentieth-century admiral had a horizon that embraced his entire fleet.

When we speak of objects that are seen or not seen we have in mind those things that endure and are therefore represented by worldlines. Hence the horizon riddle applies to the particle horizon of the universe. Let us consider the particle horizon in the static Newtonian universe and show that the riddle has a simple answer. All stars were born, we suppose, the particle horizon: 10 billion light years each other and overlap. Suppose the billion light years of information light, and this takes 10 years to reach A: B information traveled away. A billion years ago when the universe was a distance of 4 billion light years old, we see A’s horizon at that time. As the universe expanded, A’s horizon is receive the aid of Figure 19.6. What if neither B nor A’s horizon were a real horizon? Thus a real horizon is gained from beyond what lies beyond.

Cosmic horizons. The horizon is not resolved; when i
born, we suppose, 10 billion years ago, and the particle horizon is hence at a distance of 10 billion light years. Observers A and B see each other and have similar horizons that overlap. Suppose that B is at a distance of 6 billion light years from A. Observer B sends out information that travels at the speed of light, and this information takes 6 billion years to reach A. Therefore, A receives from B information that was sent 6 billion years ago when the luminous universe was 4 billion years old. B's particle horizon in the past, when the information was sent, was at a distance of 4 billion light years. Hence B's horizon at that time does not extend beyond A's horizon. Actually, B's horizon in the past, when information was sent, just touches A's horizon at the time that information is received. With this argument, and the aid of Figure 19.6, it is apparent that neither B nor any other observer can extend A's particle horizon. The particle horizon is thus a real horizon, and information cannot be gained from other observers concerning what lies beyond this horizon.

Cosmic horizons are information barriers. The horizon riddle is usually easily resolved; when it cannot be resolved, as in the case of ships at sea, then we are not talking about true horizons that are information barriers.

"A SLOW SORT OF COUNTRY"

HUBBLE SPHERE. Static universes are not realistic, and horizons in static universes, though illustrative, are not very important. In a preambling fashion we consider here what happens in an expanding universe. A more precise treatment follows in subsequent sections.

In earlier chapters we have encountered briefly the possibility of horizons in expanding universes and have occasionally referred to the "observable universe." According to the velocity-distance law of the universe the recession velocities of galaxies increase with distance. If the distance is doubled the recession velocity is doubled. The recession velocity eventually equals the velocity of light, and this happens at the Hubble length denoted by \( L = c/H \), where \( H \) is the Hubble term. The Hubble length is not known precisely; we have assumed that it has a value of 15 billion light years. Galaxies beyond the Hubble length are receding faster than the velocity of light. The velocity-distance law of the universe applies everywhere in space at an instant in time; it tells us how fast galaxies are receding at the present moment, and we must remember that their recession velocities were different at the time they emitted the light that we now receive.

We are at the center of the Hubble sphere, a sphere that has a radius equal to the Hubble length, and has therefore a radius of 15 billion light years. Inside the Hubble sphere are all the galaxies that at present recede more slowly than the velocity of light, and outside the sphere are all the galaxies that at present recede faster than the velocity of light. In earlier chapters we have rather crudely identified the Hubble sphere with the observable universe. But the observable universe is bounded by the particle horizon, and this, as we shall see, is...
not at the edge of the Hubble sphere. Indeed, if the Hubble sphere and the observable universe were of the same size, the observable universe would be infinitely large in a static universe. But static universes, with their infinitely large Hubble spheres, have particle horizons at finite distance, and therefore in general the Hubble sphere is not the same as the observable universe.

COUNTRY OF THE RED QUEEN. Consider a galaxy outside the Hubble sphere that emits a ray of light in our direction. We are the observer, as shown in Figure 19.7, at the center of the Hubble sphere. The lightray travels through space at the velocity of light, but the space through which it moves is receding from us faster than light. Although the lightray hurries toward us it is actually receding. As Eddington said: “Light is like a runner on an expanding track with the winning-post receding faster than he can run.” The edge of the Hubble sphere is thus a kind of horizon. All lightrays emitted in our direction within the Hubble sphere approach us, whereas all lightrays emitted outside the Hubble sphere recede from us. The lightrays emitted in our direction at the edge of the Hubble sphere stand still; they hurry toward us at the same velocity as that at which expanding space carries them away. The edge of the Hubble sphere is the country of the Red Queen: “Now, here, you see, it takes all the running you can do, to keep in the same place,” said the Red Queen to Alice.

This picture of a horizon in an expanding universe is simple to understand but unfortunately is misleading. We have viewed everything in space at the same instant – in a sort of snapshot picture – and have not allowed for the important fact that the observer sees things as they were in the past and cannot see them as they are now at great distances. It is true that galaxies outside the Hubble sphere recede faster than light can travel, and the lightrays they send in our direction are also receding, but this does not mean that all galaxies beyond the Hubble sphere are necessarily always hidden from view.

The rate of expansion of most universes is not constant. In a decelerating universe in which the Hubble term decreases with time, the Hubble length steadily increases, and the Hubble sphere consequently expands. Lightrays outside the Hubble sphere, moving in our direction, may therefore eventually be overtaken and lie inside the Hubble sphere. These rays will at last begin to approach us, and later we shall receive them. On an expanding track the runner sees the winning-post receding, but the runner must keep running and not despair because the expansion of the track may be slowing down, and the winning-post in that case can ultimately be reached. This argument enables us to realize that the Hubble sphere is not necessarily the observable universe.

DISTANCES. Before discussing horizons in more detail we should make clear what we mean by distance. Cosmologists use various
kinds of distance indicators, such as the luminosity distance and the distance by apparent size, but we shall consider only "pure" distances of the kind displayed in spacetime diagrams.

We look out in any direction in the expanding universe and see galaxies of increasing redshift. It is sometimes said we see galaxies of increasing redshift stretching away to greater and greater distances. Do we? The answer is yes if we mean their present distances as used in the velocity-distance law. But the answer is emphatically no if we mean their distances at the time they emitted the light that is now seen. We cannot see galaxies at their present distances; we see them only at the distances they had in the past when they emitted the light now seen, and these distances do not continually increase with redshift in a big bang universe.

We therefore have in mind two kinds of distance when we speak of a distant galaxy: its present distance and its distance at the time of emission of the light we now see. The first will be called the reception distance (the distance at the time of reception of light) and the second the emission distance (the distance at the time of emission of light). These two distances have the simple relation

$$1 + z = \frac{\text{reception distance}}{\text{emission distance}}$$

where $z$ is the observed redshift. The reception distance is always greater than the emission distance in an expanding universe.

We have asked whether distance increases with redshift. To answer this question we must look at the spacetime diagram, and the conclusion we shall reach is anticipated by the following remarks. The reception distances of galaxies in an expanding universe always increase with redshift, and the larger their redshifts, the larger are their reception distances. The emission distances at first also increase with redshift, and then a remarkable thing happens: The emission distances stop increasing with redshift and begin to decrease! Faint galaxies of large redshifts are actually closer to us at the time of emission than are brighter galaxies of smaller redshifts. This strange situation, unlike any encountered by Alice in her adventures, happens in all expanding universes for which the deceleration term $q$ is greater than $-1$. It does not happen in the de Sitter and steady state universes for which $H$ is constant and $q$ is equal to $-1$.

A spacetime diagram convenient for our purpose is shown in Figure 19.8, in which worldlines diverge in all directions from a big bang. Space is represented by any spherically curved surface that is perpendicular to the worldlines, and time is measured radially along the worldlines. Each worldline is a galaxy fixed in space, and as time advances, the galaxies recede from each other because of the expansion of space. An expanding balloon is a helpful analogy; galaxies are denoted by points marked on the surface of the balloon, and as the balloon is inflated, the "galaxies" recede from each other. The two-dimensional surface of the balloon represents our three-dimensional space, and the radial direction represents time and should not be confused with the third dimension of space.

One of the worldlines — it does not matter which — is chosen as the observer's and

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**Figure 19.8.** A big bang with worldlines diverging in all directions. Do not let this diagram mislead you into thinking that the big bang occurs at a point in space. Time is measured along the worldlines, and space is represented by any spherically curved surface perpendicular to the worldlines.
labeled \( O \), as in Figure 19.9. The observer’s lightcone at a certain instant (call it “now”) stretches back and intersects other worldlines, such as those of \( X \) and \( Y \). Because of the expansion of space the lightcone does not stretch out straight, as in a static universe, but instead is curved and reaches back into the big bang. The observer is thus able to look back into the big bang, and the lightrays received from the big bang, which travel forward along the lightcone, comprise the cosmic radiation. On the lightcone are events that emit light, and the closer the events are to the big bang, the larger is their observed redshift. Hence redshift increases steadily as we proceed along the lightcone toward the big bang.

Figure 19.9 shows the emission and reception distances of galaxy \( X \); the emission distance is the distance in space of \( X \) at the time \( O \) receives the lightrays. The reception distance of the second galaxy \( Y \) is greater than that of \( X \), and \( Y \) at present (at the moment “now”) is further away than \( X \). \( Y \)’s worldline intersects the lightcone at a point closer to the big bang, and its redshift, as seen by \( O \), is greater than that of \( X \). From this it is clear that redshift increases with reception distance, and galaxies of large redshifts are at present further away than galaxies of small redshifts.

As we proceed back along the lightcone the emission distance at first increases, then finally reaches a maximum value, and thereafter begins to decrease. \( Y \)’s emission distance in Figure 19.9 is less than that of \( X \), although \( Y \) is now further away and has the greater redshift. In the Einstein–de Sitter universe the maximum emission distance is \( 8/27 \) of the present Hubble length, almost 5 billion light years, and occurs at a redshift of \( 5/4 \).

Galaxies at maximum emission distance have a recession velocity equal to the velocity of light. They were at the edge of the observer’s Hubble sphere at the time of emission. Galaxies now observed with redshifts less than that corresponding to the maximum emission distance had recession velocities less than the velocity of light at the time of emission (and were inside the Hubble sphere), and those with larger redshifts had recession velocities greater than the velocity of light at the time of emission (and were outside the Hubble sphere). Notice in Figure 19.9 that when \( Y \) emits light toward \( O \) the lightcone is diverging away from \( O \)’s worldline; the lightrays leaving \( Y \) at first move away from \( O \) and then, after reaching maximum emission distance, converge on \( O \).

PARTICLE HORIZONS

RECEDING PARTICLE HORIZON. We recall that beyond the particle horizon are galaxies that cannot be observed. In more technical language we say that beyond the particle horizon are worldlines that do not intersect the observer’s lightcone. Worldlines that do...
intersect the observer's lightcone represent galaxies within the observable universe and are visible at some state in their evolution, whereas worldlines that do not intersect the observer's lightcone represent galaxies outside the observable universe and at the time of observation are not visible at any stage in their evolution.

The type of spacetime diagram shown in Figures 19.8 and 19.9 is useful for illustrating the nature of the particle horizon in an expanding universe. In such a diagram, as in Figure 19.10, we notice immediately that space "now" is divided into two regions: the first, surrounding the observer, contains all worldlines that intersect the observer's lightcone; the second, further away from the observer, contains all worldlines that do not intersect the lightcone. The first region is the observable universe, the second region is the unobservable universe, and the two are separated by the particle horizon. The distance of the particle horizon is measured in the observer's present space (at time "now") and is the recession distance of galaxies of infinite redshift.

We consider next a later instant, labeled "later" in Figure 19.11, when the universe has expanded further. It is clear that the observer's lightcone now intersects more worldlines and the particle horizon has receded to a greater distance. Because the particle horizon has receded, the observable universe has expanded. We notice that the observable universe expands faster than the actual universe, and as the particle horizon recedes, it sweeps out faster than the receding galaxies. In fact, the particle horizon sweeps past the galaxies at the velocity of light. This is a general rule: the observable universe overtakes the galaxies at the velocity of light.

In the Einstein–de Sitter universe, with which it is simple and convenient to work, the particle horizon is always at twice the Hubble length. The observable universe, in other words, has a radius twice that of the Hubble sphere. The recession velocity of the galaxies at the edge of the observable universe, from the velocity–distance law, is hence twice the velocity of light, and the edge of the observable universe – which is the particle horizon – is moving away at three times the velocity of light. The situation is similar, but not exactly the same, in the other two Friedmann universes at the present stage of cosmic evolution.

UNIVERSES WITHOUT PARTICLE HORIZONS.
Some universes, for example, the Milne, de
Sitter, and steady state universes, do not have particle horizons. All worldlines intersect an observer’s lightcone and all galaxies in the universe are visible. To show diagrammatically why such universes are possible, we use a different type of spacetime. This spacetime, as shown in Figure 19.12, consists of comoving rather than ordinary space. All comoving objects have constant comoving distances, and in this new spacetime all worldlines are parallel. An observer’s lightcone again does not reach out straight, as in a static universe; in this case it opens up and spreads out, as shown. In many universes, such as the Friedmann kind, the lightcone reaches back to the beginning of time at a finite comoving distance, and a particle horizon exists at the worldline \(X\). In some other universes, however, such as the Milne universe (in which \(R\) increases at a constant rate), the lightcone reaches the beginning of time at an infinite comoving distance, and consequently there is no particle horizon. The observable universe fills the entire actual universe and all galaxies are visible. The de Sitter and steady state universes are of this kind, but are more complicated, and will be considered when we discuss event horizons.

**SPACETIME DIAGRAMS WITH STRAIGHT LIGHTCONES.** The time has come to introduce the reader to a useful trick when discussing horizons of any kind. We have just used a spacetime diagram (Figure 19.12) in which comoving space takes the place of ordinary space. This type of diagram has the advantage that all worldlines are parallel to one another, but retains the disadvantage that the lightcone spreads out in a way so peculiar that often it is difficult to realize what is happening. Why not—as the next logical step—alter time so that the lightcone is straight, as in a static universe? We then have a spacetime diagram of comoving space and “altered time” that looks like an ordinary spacetime diagram of a static universe. And we know how to handle horizons in a static universe.

The spacetime interval between two events close together is given by

\[(\text{spacetime interval})^2 = (\text{time interval})^2 - (\text{space interval})^2\]

in which space intervals are measured in light travel time. All lightrays follow paths—technically known as null geodesics—for which the spacetime intervals are zero. Hence, for a lightray and anything else that travels at the speed of light,

\[(\text{time interval})^2 = (\text{space interval})^2 - (R \times \text{comoving space interval})^2\]

If we now change the intervals of time, such that

\[\text{altered time interval} = \frac{\text{time interval}}{R}\]

light rays will follow paths determined by

\[(\text{altered time interval})^2 = (\text{comoving space interval})^2\]

and hence

\[\text{altered time interval} = \pm \text{comoving space interval}\]

(the plus sign is for the forward lightcone and the minus sign for the backward lightcone). This last relation between intervals of
altered time and comoving space is for a spacetime, like that shown in Figure 19.13, in which the lightcones are straight, as in a static universe. The advantage of this type of diagram is that it allows us to treat horizons the way we do in a static universe. There are four possibilities that must be considered.

When a universe has a beginning and an ending in altered time, the spacetime diagram is of the type shown in Figure 19.14. The closed Friedmann universe that begins and ends with a bang belongs to this class.

When a universe has a beginning but no ending in altered time, the spacetime diagram has a lower boundary but no upper boundary, as shown in Figure 19.15. The Einstein–de Sitter universe and the Friedmann universe of negative curvature that begin with bangs and expand forever are examples in this class.

Figure 19.13. A spacetime diagram that consists of comoving space and "altered time." If we straighten out the lightcone by altering the intervals of time, the spacetime diagram of a nonstatic universe looks like that of a static universe. This allows us to study horizons in nonstatic universes just as in static universes.

Figure 19.15. A spacetime diagram of comoving space and altered time that has a beginning but no ending. The particle horizon is at the worldline X, and there is no event horizon.

Figure 19.14. A spacetime diagram of comoving space and altered time that has a beginning and an ending. The worldline X is at the particle horizon. Note also that there is an event horizon.

Figure 19.16. A spacetime diagram of comoving space and altered time that has an ending but no beginning. There is an event horizon but no particle horizon.
When a universe has an ending but no beginning in altered time, the spacetime diagram has an upper boundary and no lower boundary, as shown in Figure 19.16. The de Sitter and steady state universes are examples in this class.

Finally, there are some universes, such as the Einstein static universe and the Milne universe, that have neither beginnings nor endings in altered time, as shown in Figure 19.17.

It is now easy to see which universes have particle horizons. The necessary condition for a particle horizon is that altered time have a beginning, as in Figures 19.14 and 19.15. The lightcone in this case stretches back; terminates at the lower boundary, where the universe begins; and fails to intersect all worldlines. Many universes, including the Friedmann universes, have particle horizons. When, however, altered time has no beginning, and there is no lower boundary, as in Figures 19.16 and 19.17, the lightcone stretches back without limit and intersects all worldlines in the universe. In this case there is no particle horizon.

By straightening out the lightcone (which is always possible when we know how the scaling factor changes with time), we make it immediately obvious whether a particle horizon exists: There is a particle horizon whenever altered time has a beginning.

As the observer's lightcone advances into the future, and therefore advances also in altered time, the particle horizon recedes and more and more galaxies become visible. Once a worldline is within the particle horizon - and hence within the observable universe - it always remains within the particle horizon. As Figure 19.18 shows, galaxies can never move out of the observable universe; those now observed will remain always observable.

**EVENT HORIZONS**

**THE ULTIMATE LIGHTCONES.** Beyond the event horizon lie events that an observer is never able to see. With our new spacetime diagrams, consisting of comoving space and altered time, event horizons are easy to understand. Consider an observer's worldline O and an event labeled a that is somewhere in spacetime, as in Figure 19.19. O's lightcone advances into the future, and when it reaches the moment labeled "later,"

![Figure 19.17](image.png)

*Figure 19.17. A spacetime diagram of comoving space and altered time that has no beginning and no ending. There are no particle and event horizons.*

![Figure 19.18](image.png)

*Figure 19.18. As the moment o of observation advances up the observer's worldline O, the particle horizon recedes. Once a worldline such as Z is within the particle horizon, it remains inside forever. This means that a galaxy inside the observable universe will remain always inside and therefore always observable while it exists.*
beyond the upper limit, and hence there are events that can never lie on \( O \)'s lightcone and will never be observed. The necessary condition for an event horizon is that altered time have an ending; the event horizon is nothing more than the observer's ultimate lightcone at the end of altered time. All those events inside the ultimate lightcone, or event horizon, are at some time observed, and all those events outside are never observed.

**BLUESHIRTS AND REDSHIFTS AT EVENT HORIZONS.** Universes that collapse into big bangs, such as the closed Friedmann universe, always have event horizons. Consider an observer, of worldline \( O \), who is in a collapsing universe. At some instant \( o \) the observer receives signals from an event \( a \), as shown in Figure 19.21. The redshift of the received lightrays is given by

\[
z = \frac{R_0}{R} - 1
\]

where \( R_0 \) is the value of the scaling factor at the time of reception and \( R \) is the value at

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**Figure 19.21.** All collapsing universes that terminate in big bangs have an end in altered (and ordinary) time, and therefore possess event horizons. At moment \( o \), observer \( O \) sees event \( a \) and sees it blueshifted. As the moment of observation \( o \) approaches the end, events are seen with increasing blueshift, and at the last possible moment, all events on the lightcone are seen to happen infinitely rapidly. In this case the event horizon has maximum blueshift.
the time of emission. Because the universe is collapsing, $R_0$ is less than $R$, and the redshift is negative. A negative redshift means that light is blueshifted. As O’s last moment approaches, and the point $o$ moves toward the end, the value $R_0$ of the scaling factor shrinks, and all events close to the event horizon have large blueshifts. At the event horizon itself the blueshift is maximum (the redshift has the minimum value of $-1$), and all events are seen to happen infinitely rapidly.

There are universes of the whimper kind — they expand forever and do not terminate in big bangs — that also have endings in altered time. The de Sitter and steady state universes are of this kind and therefore have event horizons. By repeating the argument made previously we find the events close to the event horizon have large redshifts. This is because the value $R_0$ of the scaling factor at the time of observation is always greater than the value $R$ at the time of emission, and as the point $o$ approaches the end of altered time, the ratio $R_0/R$ becomes large and approaches infinity.

An alternative way of looking at the de Sitter and steady state universes is to consider a spacetime diagram that consists of comoving space and ordinary time, as in Figures 19.12 and 19.22. The observer O at the moment o sees event a. As the moment of observation o advances into the unlimited future, the lightcone moves upward and approaches the event horizon more and more slowly. Close to the event horizon observed events happen very slowly, and at the event horizon they are frozen into an immobility of infinite redshift.

**DE SITTER AND STEADY STATE UNIVERSES.**

The de Sitter universe is at first puzzling, and it must be admitted that our comments so far have not greatly clarified what happens. The Hubble term $H$ is constant and the Hubble sphere has a radius that is always the same. At the edge of the Hubble sphere the recession velocity equals the velocity of light, and calculations show that in the de Sitter universe the redshift at the Hubble length is infinite. The Hubble sphere is therefore the observable universe. But why is the edge of the Hubble sphere an event horizon and not a particle horizon? Consider also this problem: As the universe expands the observed galaxies are carried further and further away from the observer and become progressively more redshifted; what happens to these galaxies — do they finally cross the edge of the Hubble sphere and disappear from sight?

In the de Sitter universe the edge of the Hubble sphere is a true horizon. Yet it is not a particle horizon because at any instant all galaxies in the universe are visible to an observer. Any galaxy now beyond the Hubble sphere, no matter how far away, has part of its worldline inside the Hubble sphere and is therefore observable. The galaxies recede and move out of the Hubble sphere, and yet, oddly enough, the observer never sees them crossing the Hubble edge.

The spacetime diagram that consists of comoving space and altered time has shown us that the de Sitter universe has an event horizon but no particle horizon. To under-
Figure 19.23. The de Sitter universe displayed in ordinary space and ordinary time. The Hubble sphere has constant radius about the observer's worldline O. All worldlines diverge away from O, and every worldline at some time in the past intersects the edge of the Hubble sphere. The observer's lightcone curves back, as shown, and approaches asymptotically the edge of the Hubble sphere. The observer sees all worldlines, and hence there is no particle horizon. Because events outside the Hubble sphere are never observed, the edge of the Hubble sphere is the event horizon.

Figure 19.24. The steady state expanding universe is different from the de Sitter universe only in that galaxies are continually created so as to maintain a constant density. If we are to consider only worldlines of luminous galaxies, a worldline begins when a galaxy is born and terminates when it dies. Most galaxies, such as X, are born outside the Hubble sphere and are never seen; some galaxies, such as Y, are born inside the Hubble sphere and may die before they reach the Hubble edge; and other galaxies, such as Z, cross the Hubble edge while luminous. As in the de Sitter universe, the number of galaxies of infinite redshift at the event horizon is infinitely great. There are hence an infinite number of galaxies crowded at the edge of the Hubble sphere, and the observable universe contains an infinite number of galaxies.

To understand the nature of the horizon it is more convenient to revert to a diagram of ordinary space and ordinary time, as in Figure 19.23. The Hubble sphere is of constant radius, as shown, and all worldlines of galaxies diverge away from the observer's worldline O. The observer's lightcone is curved and approaches asymptotically the edge of the Hubble sphere. The lightcone consequently intersects the worldlines of all galaxies in the universe, and yet never extends beyond the Hubble sphere. Because all worldlines are intersected by the observer's lightcone, and hence all galaxies are observed, there is no particle horizon. The edge of the Hubble sphere is an event horizon because it is the observer's ultimate lightcone, and all events outside the Hubble sphere are never observed.

Figure 19.23 makes it apparent that the observer sees the galaxies approaching the edge of the Hubble sphere, with increasing redshift, but never sees them crossing the edge. At the edge are crowded an infinite number of galaxies of infinite redshift.
Galaxies, of course, do not shine forever, and their worldlines as luminous sources are therefore of finite length. This is something for the reader to think about, and suitable amendments, where necessary, can easily be made in what has been previously said. Worldlines of finite length are of particular interest in the steady state universe.

The steady state universe expands in the same way as the de Sitter universe and has an event horizon at the edge of the Hubble sphere. Galaxies, however, are continually created everywhere in order to maintain a constant density, and hence it is not true to say that all galaxies are observable in the steady state universe. Most galaxies are created outside the observer's Hubble sphere and are never seen at any time, as indicated by worldline X in Figure 19.24. Also, a galaxy born inside the Hubble sphere may die and become nonluminous before it has reached the Hubble edge, as indicated by worldline Y. Even so, the number of luminous galaxies that cross the Hubble edge, and have worldlines such as Z, is still infinite. At the event horizon, where the redshift is infinite, and where light rays that try to reach us stand still, there is an infinite number of galaxies.

REFLECTIONS

1 "A horizon is here defined as a frontier between things observable and things unobservable. Two quite different types of horizon exist which are here termed event-horizon and particle-horizon" (Wolfgang Rindler, "Visual horizons in world-models," 1956).
   • "God not only plays dice. He also sometimes throws the dice where they cannot be seen" (Stephen Hawking, 1975).
2 Do we distinguish between events (happenings) and worldlines (objects) in ordinary language?
   • Explain particle and event horizons. What would happen if light traveled infinitely fast?
   • In a static universe of stars that have been shining for a past eternity there is no particle horizon; if the stars shine for a future eternity there is no event horizon. Explain.
   • Discuss the horizon riddle.
   • Although cosmic horizons form one of the most fascinating topics in cosmology, they are usually not discussed in elementary texts. On occasion they are referred to briefly, and then it is implied that the edge of the Hubble sphere is a horizon. Discuss the difference between the Hubble sphere and the observable universe. When are they the same?
   • In which direction do galaxies cross the particle horizon?
   • As time passes, the observable universe contains more and more galaxies; is this true also in a collapsing universe?
   • At death each person has a horizon. What kind of horizon?
   • Consider the dark night sky paradox in the Newtonian universe. What kind of horizon prevents an observer from seeing a sufficient number of luminous stars to cover the sky?
   • Once we understand horizons in a static universe, why is it then relatively easy to understand them in nonstatic universes?
   • Give examples of event horizons at which (a) the redshift z is infinite, (b) z = 0, (c) z = -1.
   • Discuss the maximum distance that can be seen in an expanding big bang universe. Could we argue that this maximum distance is a special type of event horizon?
   • How is it possible in an Einstein-de Sitter universe for a source of redshift z = 2 to be at an emission distance less than that of a source of z = 1?
   • Can you think of cosmic horizons that might exist because of the observer's forward lightcone? Such horizons determine the observer's ability to influence events and particles elsewhere in the universe. Use Figures 19.14-17 with forward lightcones.
3 Consider all big bang universes in which the scaling factor R is proportional to t^n, where t is the age of a universe and n is a
constant number less than 1. In such universes

\[
\text{particle horizon distance} = \frac{L}{1 - n} \quad (19.1)
\]

and this is the radius of the observable universe. In this expression \( L \) is the Hubble length and is the radius of the Hubble sphere. It is seen that when \( n \) is greater than \( \frac{1}{3} \), the observable universe is larger than the Hubble sphere. According to the velocity-distance law, the recession of galaxies at the particle horizon is now given by

\[
\text{recession velocity at particle horizon} = \frac{cn}{1 - n} \quad (19.2)
\]

where \( c \) is the velocity of light. The recession of the particle horizon itself, however, is given by the relation

\[
\text{velocity of particle horizon} = \frac{c}{1 - n} \quad (19.3)
\]

and this is the velocity of expansion of the observable universe. Note this relation: recession velocity at particle horizon + velocity of light = velocity of particle horizon. The particle horizon sweeps out past the galaxies at the velocity of light. In the Einstein-de Sitter universe of \( n = \frac{1}{3} \), the particle horizon is at a distance \( 2L \) (30 billion light years), the recession velocity of galaxies at the particle horizon is \( 2c \) (twice the velocity of light), and the horizon itself moves away at \( 3c \) (thrice the velocity of light). Discuss what happens in the Dirac universe of \( n = \frac{1}{3} \).

When \( n \) is equal to or greater than unity, there is no particle horizon, and the observer sees all luminous objects in the universe. Thus, in Milne's universe of \( n = 1 \), there is no particle horizon. Milne disliked universes with horizons and regarded the absence of a horizon in his universe as a distinct advantage.

The maximum distance of the lightcone from an observer's worldline is expressed by

\[
\text{maximum emission distance} = L \ln^{1/(1-n)} \quad (19.4)
\]

and the redshift of sources at this distance is given by

\[
\text{redshift at maximum emission distance} = n^{-V/(1-n)} - 1 \quad (19.5)
\]

---

*Figure 19.25.* \( X \) and \( Y \) are the worldlines of two objects at equal distances in opposite directions from observer \( O \). The time required for light to travel from \( X \) to \( O \), and from \( Y \) to \( O \), is denoted by \( T \). At the moment "now," when \( O \) observes \( X \) and \( Y \), both \( X \) and \( Y \) have just begun to see each other for the first time if the universe has an age of \( 3T \). In this case the distance of \( X \) and \( Y \) from \( O \) is one-third the particle horizon distance. When the universe is younger than \( 3T \), \( O \) is able to see \( X \) and \( Y \) but \( X \) and \( Y \) cannot see each other.
Thus, when \( n = \frac{1}{3} \), the maximum emission distance is \( \frac{4}{3} L \), and sources at this distance have redshift \( z = 1 \). When \( n = \frac{1}{3} \), as in the Einstein--de Sitter universe, the maximum emission distance is \( 8/27 \) times \( L \), and the corresponding redshift is \( 5/4 \). The recession velocity of sources at maximum emission distance, at the time of emission, is always equal to the velocity of light; these sources are therefore at the edge of the observer’s Hubble sphere at the time they emit the light that is now seen.

4 Two objects are at equal distances in opposite directions from us, as shown by worldlines \( X \) and \( Y \) in Figure 19.25. We are able to see these objects, but can they see each other? Let \( T \) be the time (in units of altered time) that light takes to travel to us from \( X \) and \( Y \). The time that light takes to travel from \( X \) to \( Y \) and from \( Y \) to \( X \) is obviously \( 2T \). When the universe is older than \( 3T \) we observe \( X \) and \( Y \) after they observe each other for the first time; when the universe is younger than \( 3T \) we observe \( X \) and \( Y \) before they observe each other. There is thus a maximum distance beyond which the observed objects \( X \) and \( Y \) do not know that each other exists. By examining the diagram it can be seen that this maximum distance is one-third the distance of the particle horizon (this is a reception distance and applies at the present moment). Hence the answer to our question is that objects in opposite directions, further away than one-third the particle horizon distance, cannot see each other. In the Einstein--de Sitter universe this distance is 10 billion light years, corresponding to a redshift of 1.25.

This raises the interesting problem of why the universe is isotropic and homogeneous. Regions in opposite directions of sufficiently large redshift are isolated from each other; they have not influenced each other and are unaware of the other’s existence; and yet in an isotropic universe they are in identical states. This is the puzzling feature of isotropy and homogeneity: How can regions be alike when they do not know that each other exists?

![Figure 19.26. The observed universe consists of only those events that lie on the observer’s backward lightcone. A small region about the observer’s worldline contains events not on the lightcone whose existence can be inferred from the immediate environment. The history of the Galaxy, the Solar System, the Earth, and the human race is confined to this region. All the rest of spacetime contains events that at present are unobserved. If there is an event horizon, then beyond this horizon are events that can never be observed.](image)

“We are unable to obtain a model of the universe without some specifically cosmological assumptions which are completely unverifiable” (George Ellis, “Cosmology and verifiability,” 1975). The problem is that we observe isotropy, which we cannot explain, and we assume homogeneity, which we cannot verify. We observe only those events that lie on our backward lightcone, as in Figure 19.26, and the rest of spacetime – except for a small region about our worldline – is unobserved.

**SOURCES**

Centrella, J. C. “Visual horizons in cosmological models: a study.” Senior honors thesis, Department of Physics and Astronomy, University of Massachusetts. 1975. Most of
this chapter is based on Joan Centrella's honors thesis.