PHYS 8401: Methods of Mathematical Physics F-22

(Mathematical Problem Solving for Physics)

Instructor: K. Nakayama, Room 219

Text: *Mathematical Methods for Physicists*, 7th Ed., G. B. Arfken, H. J. Weber and F. E. Harris (Academic Press, 2013).

Reference: *Mathematical Methods of Physics*, 2nd Ed., J. Mathews and R. L. Walker, (Addison Wesley, 1970).

Office Hours: **02:30pm-03:30pm,** Thursdays, or by appointment (using zoom).

Homework: Emphasis of Course! Problems will be collected and selected problems graded.

Grade: Total Grade = 100% Homework.

The letter grade will be assigned as:

$$\begin{array}{rcl} & A \equiv [90,100] & A- \equiv [87,90) \\ B+ \equiv [80,87) & B \equiv [75,80) & B- \equiv [70,75) \\ C+ \equiv [65,70) & C \equiv [60,65) & C- \equiv [55,60) \\ D \equiv [50,55) & F \equiv [0,50) & \end{array}$$

Standard rounding will be used for the final numerical grade. For example, 89.4999 will be 89 and A-, but 89.5 will be 90 and A. There are no exception to these letter grade assignments. All withdrawal will be processed in accordance with University policy as stated in the undergraduate bulletin. For withdrawals before the midpoint, a grade of "W" will be assigned for all cases.

Tentative Topics

Below is a tentative list of topics to be covered. Any change to it will be announced in class. You are responsible to keep track on such changes by attending class.

1. Functions of a Complex Variable:

   (a) Complex Numbers

   (b) Functions of a Complex Variable

   (c) Analytic Functions and Singularities

   (d) Integrals of Functions of a Complex Variable

   (e) Taylor and Laurent Series

   (f) Residue Theorem

   (g) Contour Integration and Evaluation of Definite Integrals

2. Fourier Series and Transforms:

   (a) Fourier Series

   (b) Fourier Transforms

   (c) Dirac Delta Function

   (d) Convolution Theorems

   (e) 3–D Fourier Transforms

   (f) Laplace Transforms

   (g) Applications

3. Ordinary Differential Equations:

   - Terminology

   (a) Linear 2nd order ODE's $\rightarrow$ HDE's and IDE's

   (b) Superposition Principle, Linear Independence and Wronskians

   (c) General Approach

   - Variation of Parameters:

   (a) A Second Soln. to HDE

   (b) Gen. Soln. to IDE

(c) Example (Green's fctn.)

- Series Solutions to 2nd order, Linear HDE

(a) Regular and Singular Points and Types of Solns.

(b) Bessel's DE and Functions

(c) Legendre's DE and Polynomials

4. Special Functions:

- Legendre Functions

(a) Representations: ODE, Explicit Form, Rodrigue's Formula, Generating Function

(b) Recursion Relations

(c) Particular Values, Othogonality and Normalization

(d) Legendre Functions of the 2nd Type

(e) Associated Legendre Functions: First and Second Types

(f) Spherical Harmonics

(g) Spherical Harmonic Addition Theorem : Derivation and Application

- Bessel Functions

(a) Generating Function and Recurrence Relations

(b) Asymptotic Forms

(c) Orthogonality

(d) Fourier-Bessel Series and Hankel Transforms

(e) Expansion of a Plane Wave into Cylindrical Waves

(f) Spherical Bessel Functions

    i. Recurrence and Orthogonality

    ii. Role in 3-D Fourier Transforms

    iii. An application

(g) Modified Bessel Functions: Cylindrical and Spherical

5. Partial Differential Equations:

   (a) Boundary Conditions, Green's Identities and Uniqueness

   (b) Separation of Variables: Introduction through the 3-D wave Equation

   (c) Helmholtz Equation in Cartesian Coordinates

      i. Example: Diffusion of Temperature Inside a Cube

   (d) Helmholtz and Laplace Equations in Spherical Coordinates (Examples)

      i. Electrostatic Potential of a Point Charge Near a Grounded Conducting Sphere

      ii. Electrostatic Potential of a Uniformly Charged Circular Wire

      iii. Acoustic Radiation from a Split-sphere Antenna

   (e) Helmholtz and Laplace Equations in Cylindrical Coordinates (Examples)

      i. Vibrations of a Circular Drumhead

      ii. Electromagnetic Wave Scattering from an Infinitely Long Cylindrical Conductor

6. Inhomogeneous Boundary-Value Problems and Green's Functions:

   (a) Eigenvalue Problem in Function Space

   (b) Eigenfunctions and Green's Functions:

   (c) Sturm-Liouville Problem

   (d) Eigenfunction Expansions as Solutions to PDE's

   (e) Inhomogeneous PDE's and Inhomogeneous Boundary Conditions

   (f) Green's Function Techniques

      i. Eigenfunction expansions of G=Green's function

      ii. Physical interpretation of G

      iii. Green's function for a vibrating string

      iv. The (1-D) ODE method for getting G

v. Green's function for a vibrating membrane

vi. Green's functions in Electrodynamics

vii. Green's function for Helmholtz' Eq. in spherical coordinates