The University of Georgia
Department of Physics and Astronomy
Graduate Qualifying Exam — Part II
January 6, 2020

Instructions:

- Start each problem on a new sheet of paper. Write the problem number on the top left of each page and your pre-arranged prelim ID number (not your name) on the top right of each page. Leave margins for stapling and photocopying.

- Put your answers only on one side of the paper; do not write on the back side.

- If not instructed otherwise, derive the mathematical solution for a problem from basic principles or general laws (Newton's laws, Maxwell equations, Schrödinger equation, laws of optics, laws of thermodynamics, etc.). Use common mathematical symbols and avoid writing lengthy texts.

- No formula sheets are allowed.

- You may use a calculator for basic operations only (i.e., not for referring to notes stored in memory, symbolic algebra, symbolic and numerical integration, etc.). The use of cell phones, tablets, and laptops is not permitted.

Part II contains problems 6 - 10.

Modern Physics

Problem 6:

The Spitzer Space Telescope was launched in 2003 to detect infrared radiation. Suppose a particular detector on the telescope is sensitive over part of the near-infrared region of wavelengths 980 to 1920 nm. Astronomers want to detect the radiation being emitted from a red giant star and decide to concentrate on wavelengths from the Paschen series of the hydrogen atom. You may use $R_H = 1.096776 \times 10^7 \text{m}^{-1}$. (3 parts)

(1) What are the known wavelengths in this wavelength region?

(2) The detector measures wavelengths of 1334.5, 1138.9, and 1046.1 nm believed to be from the Paschen series. Why are these wavelengths different from those found in part 1?

(3) How fast is the star moving with respect to us?

Problem 7:

Consider a distant galaxy emits a strong line emission at frequency, f_{src}, as measured in the rest frame of the source. Imagine an observer (Q) at rest measures the frequency of the emission as f_{obs}. The galaxy is receding from the observer at radial velocity of v. If Δt is the time between the two detections of successive crests of the line emission, then the observed linear length between the two crest (i.e., wavelength) can be expressed as

$$\lambda = c\Delta t + v\Delta t.$$

(1) Using the time dilation relation ($\Delta t = \gamma \Delta t'$) in conjunction with the above equation, show that the observed frequency of the emission is

$$f_{obs} = \sqrt{\frac{1-\beta}{1+\beta}} f_{src}$$

where $\beta = v/c$ and c is the speed of light.

(2) If the galaxy is approaching to the observer, how do you need to rewrite the equation for the wavelength? Show that the observed frequency in this case is

$$f_{obs} = \sqrt{\frac{1+\beta}{1-\beta}} f_{src}$$

(3) If $f_{obs}/f_{src} = 1/3$, what is the speed of the galaxy in terms of c? Is the galaxy approaching or receding?

Quantum mechanics

Problem 8:

The ground state of a particle of mass m confined to a box between $x = -L/2$ and $L/2$ is

$$\psi(x) = \sqrt{\frac{2}{L}} \cos\left(\frac{\pi x}{L}\right).$$

(1) Calculate the ground state energy E_0.

(2) Calculate the expectation value of momentum in this state.

Problem 9:

Let $H = \hbar\omega a^\dagger a$ be the Hamiltonian for a harmonic oscillator with angular frequency ω, and write the stationary states of H as $|n>$, with n a nonnegative integer. Using the properties of creation and annihilation operators, namely, $a|n> = \sqrt{n}|n-1>$ and $a^\dagger|n> = \sqrt{n+1}|n+1>$, derive 3-dimensional matrix representations for a, $a^\dagger$, and H in the 3-dimensional Hilbert space spanned by eigenstates $|0>, |1>$, and $|2>$. Circle your final expressions for the matrices a, $a^\dagger$, and H.

<u>Thermodynamics</u>

Problem 10:

A solid cylinder with two movable pistons, each of mass M, contains a certain amount of ideal monoatomic gas. The system starts in a symmetric configuration in which the initial temperature, volume, and pressure of the gas are T_0, V_0, and p_0, respectively, and the pistons are moving towards each other with a speed u relative to the cylinder, as shown in the figure below. (4 parts)
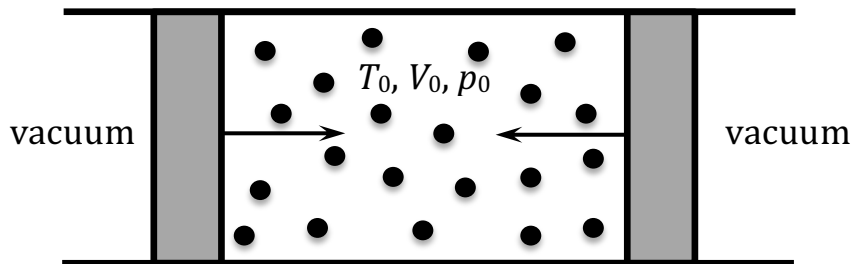
Assuming the system is thermally insulated, the process is quasi-static, that no gas is lost, and ignoring friction and the heat capacities of the pistons and the cylinder, find:

(1) the temperature, T, of the gas at the instant of maximal compression;

(2) the volume, V, of the gas at that instant.

(3) Calculate T if $T_0 = 300$ K, $V_0 = 100$ cm^3, $p_0 = 100$ kPa, $M = 100$ gram, $u = 5$ m/s.

(4) Calculate V if $T_0 = 300$ K, $V_0 = 100$ cm^3, $p_0 = 100$ kPa, $M = 100$ gram, $u = 5$ m/s.