The University of Georgia
Department of Physics and Astronomy
Graduate Qualifying Exam — Part I
January 3, 2020

Instructions:

- Start each problem on a new sheet of paper. Write the problem number on the top left of each page and your pre-arranged prelim ID number (not your name) on the top right of each page. Leave margins for stapling and photocopying.

- Put your answers only on one side of the paper; do not write on the back side.

- If not instructed otherwise, derive the mathematical solution for a problem from basic principles or general laws (Newton's laws, Maxwell equations, Schrödinger equation, laws of optics, laws of thermodynamics, etc.). Use common mathematical symbols and avoid writing lengthy texts.

- No formula sheets are allowed.

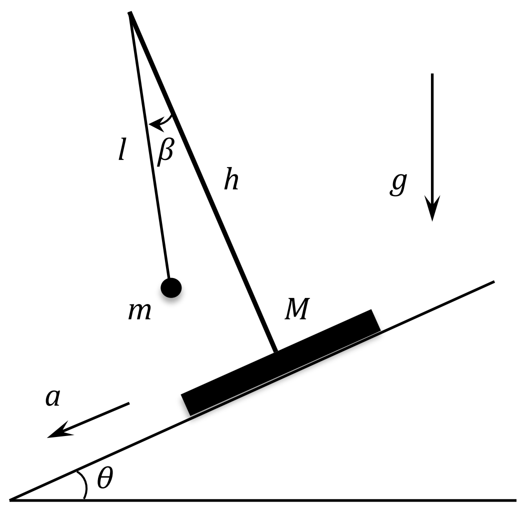
- You may use a calculator for basic operations only (i.e., not for referring to notes stored in memory, symbolic algebra, symbolic and numerical integration, etc.). The use of cell phones, tablets, and laptops is not permitted.

Part I contains problems 1 - 5.

Classical Mechanics

Problem 1:

A point mass m is suspended on an ideal massless string of length l attached to a wide heavy solid base of mass M and height $h > l$, as shown in the figure below. The system is sliding down an incline with a given acceleration a (as measured along the incline). Assume the initial conditions are chosen in such a way that the deflection angle β relative to the supporting massless rod stays fixed throughout system's motion. Ignore air resistance. (4 parts)



(1) Find β as a function of l, h, m, M, a, θ, and g (acceleration due to gravity).

(2) Assume l = 50 cm, h = 75 cm, m = 50 gram, M = 5 kg, $\theta = 30^\circ$, a = 1 m/s^2, and g = 9.81 m/s^2. What is β in this case?

(3) Assume l = 50 cm, h = 75 cm, m = 50 gram, M = 5 kg, $\theta = 30^\circ$, g = 9.81 m/s^2, and the system is sliding down in the presence of kinetic friction. The coefficient of kinetic friction between the base and the incline is $\mu_k = 0.25$. Find β in this case.

(4) Assume l = 50 cm, h = 75 cm, m = 50 gram, M = 5 kg, $\theta = 30^\circ$, g = 9.81 m/s^2, and the system is sliding down without friction. Find β in this case.

Problem 2:

Your job with NASA is to monitor satellite orbits. One day, during a routine survey, you find that a 400 kg satellite in a 1000 km high circular orbit is going to collide with a smaller 100 kg satellite traveling in the same orbit but in the opposite direction. Knowing the construction of the two satellites, you expect that they will become enmeshed into a single piece of space debris. When you notify your boss of this impending collision, she asks you to quickly determine whether the space debris will continue to orbit or crash into the Earth. What will the outcome be? (Note: The mass of the Earth is 5.97×10^{24} kg and the radius of the Earth is R_E = 6380 km.)

Electricity & Magnetism

Problem 3:

Suppose the plates of a parallel-plate capacitor move closer together by an infinitesimal distance ϵ, as a result of their mutual attraction. (3 parts)

(1) Using the equation for an outward electrostatic pressure on the surface, $P = \frac{\epsilon_0}{2} E^2$ where E is the electric field just outside the surface, express the work done by electrostatic forces, in terms of the field E, and the area of the plates, A.

(2) Express the energy lost by the field in this process. Stored energy per unit volume of the capacitor is $\frac{\epsilon_0}{2} E^2$.

(3) Comparing results from steps (1) and (2), what can you deduce?

Problem 4:

Find the electric field inside a sphere that carries a charge density that varies as $\rho(r) = kr^2$.

Optics

Problem 5:

A thin film of liquid is held in a horizontal circular ring, with air on both sides of the film. A beam of light at wavelength 550 nm is directed perpendicular onto the film, and the intensity I of its reflection is monitored. The figure gives the intensity I as a function of time t; the horizontal scale is set by t_s= 20.0 s. The intensity changes because of evaporation from the two sides of the film. Assume that the film is flat and has parallel sides, a radius of 1.80 cm, and an index of refraction of 1.40. Also assume that the film's volume decreases at a constant rate. Find that rate.