The University of Georgia
Department of Physics and Astronomy
Graduate Qualifying Exam — Part I
10 August 2017

Instructions:

• Attempt all problems. You must show your work and/or clearly explain your answers in order to be able to earn a passing grade for a problem.

• Start each problem on a new sheet of paper. Write the problem number on the top left of each page, and your pre-arranged prelim ID number (but not your name) on the top right of each page. Leave margins for stapling and photocopying.

• This is a closed-book exam. You are permitted to bring one page of notes (equations, definitions, physical constants, etc.) per exam day. You must hand in this page of notes with the exam each day.

• You may use a calculator, but only for arithmetic functions (i.e., not for referring to notes stored in memory, doing symbolic algebra, etc.).

Part I has five problems, numbered 1 — 5.
Problem 1: (one part)

Find the magnetic field of a very long solenoid, consisting of $n$ closely wound turns per unit length on a cylinder of radius $R$ and carrying a steady current $I$ (see Fig. 1 below). [The point of making the windings so close is that one can then pretend each turn is circular. If this troubles you (after all, there is a net current $I$ in the direction of the solenoid’s axis, no matter how tight the winding), picture instead a sheet of aluminum foil wrapped around the cylinder, carrying the equivalent uniform surface current $K = nI$ (Fig. 2). Or make a double winding, going up to one end and then – always in the same sense – going back down again, thereby eliminating the net longitudinal current. But in truth, this is all unnecessary fastidiousness, for the field inside a solenoid is huge (relatively speaking), and the field of the longitudinal current is at most a tiny refinement.]

Problem 2: (one part)

In the figure (Fig. 3), a glass lens is coated on one side with a thin film of magnesium fluoride (MgF$_2$) to reduce reflection from the lens surface. The index of refraction of MgF$_2$ is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550$ nm)? Assume that the light is approximately perpendicular to the lens surface.
Problem 3: (two parts)

A chain with length $l$ is held stretched out on a frictionless horizontal table, with length $y_0$ hanging down through a hole in the table. The chain is released.

(a) As a function of time, find the length that hangs down through the hole.

(b) Find the speed of the chain right when it loses contact with the table.

Problem 4: (two parts)

Find the capacitance of two concentric spherical metal shells (Fig. 4), with radii $a$ and $b$. Assume the vacuum everywhere with $\varepsilon_0$.

(a) State the definition for $C$, $V$, $Q$ for a capacitor and write down the relation between $C$, $V$, and $Q$.

(b) Find $V$ between the two shells and obtain $C$. You must show your reasoning and calculation. Writing down the answer is not sufficient.

Problem 5: (four parts)

Suppose two objects of mass $M$ are attached to two walls via springs with spring constant $\kappa$ and coupled to each other with a spring with spring constant $\kappa_{12}$ (Fig. 5).

(a) Write down the equations for the forces on each object.

(b) What are the eigen frequencies of this system?

(c) What are the equations of motion for each object?

(d) What are the normal modes of the motion?