Problem 1:

A particle of mass $m$ in the infinite square well (of width $a$) starts out in the left half of
the well, and is (at $t = 0$) equally likely to be found at any point in that region.

(a) What is its initial wave function, $\Psi(x, 0)$? (Assume it is real. Don’t forget to
normalize it.)

(b) What is the probability that a measurement of the energy would yield the value
$\pi \hbar^2 / 2ma^2$?
Problem 2:

A particle of mass $m$ is in the state

$$\Psi(x,t) = A \exp\{-a[(mx^2/h) + it]\}$$

where $A$ and $a$ are positive real constants.

(a) Find $A$.

(b) For what potential energy function $V(x)$ does $\Psi$ satisfy the Schrödinger equation?

(c) Calculate the expectation values of $x$, $x^2$, $p$, and $p^2$. 
**Problem 3:**

In the Earth’s reference frame, a tree is at the origin and a pole is at \( x = 30 \text{ km} \). Lightning strikes both the tree and the pole at \( t = 10 \mu s \). The lightning strikes are observed by a rocket traveling in the \( x \)-direction at \( 0.5c \).

**(a)** What are the spacetime coordinates for these two events in the rocket’s reference frame?

**(b)** Are the events simultaneous in the rocket’s frame? If not, which occurs first?
Problem 4:

Consider a head-on, elastic collision between a massless photon (momentum \( p_0 \) and energy \( E_0 \)) and a stationary free electron.

(a) Assuming that the photon bounces directly back with momentum \( p \) (in the direction of \(-p_0\)) and energy \( E \), use conservation of energy and momentum to find \( p \).

(b) Verify that your answer agrees with that given by Compton’s formula with \( \theta = \pi \).
Problem 5:

Figure Caption: Shown above is the energy level diagram for a hydrogen atom, showing the four lowest energy levels. Ignoring electron spin $s$, there is one independent state with energy $=-13.6 \text{ eV}$, four independent states with energy $=-3.4 \text{ eV}$, nine independent states with energy $=-1.5 \text{ eV}$, etc.

Estimate the ratio of the probability that a hydrogen atom at warm room temperature (300 K) is in one of its first excited states ($n = 2$) relative to the probability of it being in the ground state ($n = 1$). Don’t forget to take degeneracy into account. Show all steps in the calculation, even if you think that the answer will be trivial. The value of Boltzmann’s constant, $k = 8.62 \times 10^{-5} \text{ eV/K}$. 
