Polarization of light: Malus’ law, the Fresnel equations, and optical activity.

PHYS 3330: Experiments in Optics
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In this lab you will (1) test Malus’ law for the transmission of light through crossed polarizers; (2) test the Fresnel equations describing the reflection of polarized light from optical interfaces, and (3) using polarimetry to determine the unknown concentration of a sucrose and water solution.

I. POLARIZATION

You will need to complete some background reading before your first meeting for this lab. Please carefully study the following sections of the “Newport Projects in Optics” document (found in the “Reference Materials” section of the course website): 0.5 “Polarization” Also read chapter 6 of your text “Physics of Light and Optics,” by Peatross and Ware. Your pre-lab quiz cover concepts presented in these materials AND in the body of this write-up. Don’t worry about memorizing equations – the quiz should be elementary IF you read these materials carefully. Please note that “taking a quick look at” these materials 5 minutes before lab begins will likely NOT be adequate to do well on the quiz.

II. MALUS’ LAW

When completely linearly polarized light is incident on a polarizer, Malus’ law predicts the transmitted intensity to vary proportionally as the square of the cosine of the angle between the transmission axes of the analyzer and the direction of polarization of the incident light:

\[ I(\theta) = I_0 \cos^2 \theta \]

You will test this law using the polarized light of a HeNe laser, an assortment of polarizers, and a photodiode – an electrical device which generates an electric current proportional to the intensity of incident light.

III. PROCEDURE

1. Position the fixed “V” polarizer after mirror “M” to establish a vertical polarization axis for the laser light.

2. Carefully adjust the position of the photodiode so the laser beam falls entirely within the central dark square.

3. Plug the photodiode into the bench top voltmeter and observe the voltage – it should be below 200 mV; if not, you may need to attenuate the light by inserting the polarizer on a rotatable arm “R” upstream of the “V” polarizer, and rotating “R” until the reading on the photodiode is approximately 200mV.

4. Place the precision rotatable polarizer “P” just in front of the photodiode. Make sure the beam goes through cleanly.

5. Rotate “P” to maximize the transmitted power. Then rotate the polarizer so that the “0” of the vernier scale lines up with a tick mark of the fixed scale, to establish a nice place to start your systematic study of intensity versus angle.

6. Now, rotate the analyzer in steps of 4 degrees, recording the transmitted power at each step. Take data over at least 180 degrees. Estimate (and explain/justify for estimates of) the uncertainties in your measurements of voltage and the angle. (Hint: to estimate your voltage uncertainty, try monitoring the photodiode voltage for a few minutes without changing anything, to see how stable it is over a duration as long as it will take you to measure around 180 degrees....)

IV. ANALYSIS

Plot your data with error bars. Test the Law of Malus by fitting your data to the following model:

\[ P(\theta) = kV_0 \cos^2(\theta - \theta_0) + C \]  

(1)
What are the parameters $V_0, \theta_0, C$? What is $k$ and what are its units? Consider that volts is not a unit of (optical) power. To calculate the optical power in watts, you need to know that the photodiode takes every incident optical photon and converts it into one electron. This current passes over a XX Ohm resistor to produce the voltage you measure on the voltmeter. Calculate the current and you know the photons/s; calculate the energy (Joules!) per photon, and combine these results, and you know the optical power (Watts=Joules/s) incident the photodiode. Use these ideas to calculate the constant $k$ in Eq. (1).

Plot a best fit model overlaid with a plot of your data, with error bars. Plot your residuals with error bars. Report the values and uncertainties of all fitted parameters, discuss the meaning (or arbitrariness) of the values of each parameter with respect to verifying/contradicting the theory under test. Plot the fit residuals and discuss them. Report the chi-squared per degree of freedom statistic obtained by your fit, and use it to discuss the likelihood that, if the Malus theory were true, you could expect to see data such as yours. Explain any discrepancy: note – saying “we must have made an error....” is not an explanation.

Now endeavor to directly establish the values of $V_0$, $\theta_0$, and $C$ through auxiliary measurements (of your own devising). Fit your previous data to a model in which these parameters are no longer treated as free parameters of the fit, but rather are “hardwired” constants. Plot the new best fit model (overlaid with your data and the previous model). Report the values and uncertainties of all fitted parameters, discuss the meaning (or arbitrariness) of the values of each parameter with respect to verifying/contradicting the theory under test. Plot the fit residuals and discuss them. Report the chi-squared per degree of freedom statistic obtained by your fit, and use it to discuss the likelihood that, if the Malus theory were true, you could expect to see data such as yours. Explain any discrepancy: note – saying “we must have made an error...” is not an explanation.

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**V. FRESNEL EQUATIONS**

The Fresnel equations give the transmission and reflection coefficients at a dielectric interface. They depend upon the polarization and angle of incidence, and indices of refraction of the media on both sides of the interface. For light crossing from a medium of index $n_1$ to one of $n_2$, the fractional reflected intensity is predicted to be, for pure $s$-polarized light (refer to your text for the definitions of $s$ and $p$ if you are unsure).

$$R^s(\theta_i) = \frac{I_f^s}{I_i^s} = \left( \frac{n_1 \cos \theta_i - n_2 \sqrt{1 - (n_1/n_2)^2 \sin^2 \theta_i}}{n_1 \cos \theta_i + n_2 \sqrt{1 - (n_1/n_2)^2 \sin^2 \theta_i}} \right)^2$$

whereas for pure $p$-polarized light the theory predicts:

$$R^p(\theta_i) = \frac{I_f^p}{I_i^p} = \left( \frac{n_1 \sqrt{1 - (n_1/n_2)^2 \sin^2 \theta_i} - n_2 \cos \theta_i}{n_1 \sqrt{1 - (n_1/n_2)^2 \sin^2 \theta_i} + n_2 \cos \theta_i} \right)^2$$

**A. Procedure**

You will measure the reflection of polarized light off of a BK7 borosilicate glass prism. There are several steps to the alignment of the optical system.

1. Make the laser beam parallel to the surface of the table by adjusting M to achieve the same beam height close to and far from itself, marking the height of the beam on a white index card with a pen.

2. Set the angular readout of the rotational stage to 0 degrees by releasing the stage lock screw, setting the stage angle to 0 degrees, and tightening the stage lock screw again. Do not overtighten the set screw.

3. Position the prism in its mount so that the laser beam enters on the prism face marked with the white dot, and hits in the middle of the face when the face is perpendicular to the beam.

You will now make the prism face perpendicular to the table.

4. Look for the back reflection from the prism face by poking a small hole through the card at the beam height, then holding it in front of M so that the beam passes through the hole.
5. Slightly release the lock screw of the prism post holder, but not so much the post slips down. As you rotate the post back and forth, you will notice two back reflections on from the prism swinging across the card. One of the spots is an external reflection from the front surface – this is the one you want; the other is due to multiple internal reflections from the hypotenuse and opposite right side of the prism (see diagram). To differential between the two, drop a drop of methanol on the side of the prism indicated in the diagram, while observing both spots. The spot you DO NOT want will acquire ripples, which speedily vanish before your eyes (why does this happen!??)

6. To set the left-right adjustment, rotate the post in the post holder until the reflected spot is centered (possibly above or below) the hole in the card. Tighten the post holder lock screw.

7. To set the up-down adjustment, use the adjuster screws on the bottom of the prism stage. Your goal is the get the back-reflected spot centered on the hole in the note card.

You will first measure vertically polarized light – you must decide if this is $s$– or $p$– polarization!

8. Place the polarizer marked “V” about 5 in. away from M1, this establishes vertical polarization for the light incident on the prism.

9. To make your test of the Fresnel equations, rotate prism stage and measure the intensity of the reflected light as a function of the angle of incidence using the photodiode. Take data every 4 degrees or so, for angles as close to 180° and 0° practical. Remember to use the correct spot identified in step 5 above!! You will have to move the photodiode each time to be centered on the reflected spot. You must also make sure to keep your beam in the center of the face of the prism, by small adjustments to the final-bounce mirror (don’t worry – this doesn’t change your angle of incidence by much). Ensure the entire beam spot falls within the black square in the photodiode package. Read out the voltage on the voltmeter to as many digits as believe to be valid. To estimate your voltage measurement uncertainty, measure the same spot three times, each time moving the photodiode out of the beam, then back in.

Now re-take the measurements for horizontally polarized light.

10. Place the “H” polarizer about 2 inches after the “V” polarizer. You will find that not much light gets through, because the light incident upon it is mostly vertically polarized.

11. To get some light the “H” polarizer through, make use of the law of Malus – put the “R” polarizer in-between the “H” and “V” polarizers. You should now find that light is transmitted through the “H” polarizer.

12. Repeat the data taking instruction of step 9.

VI. ANALYSIS

Plot your data with error bars. Test the Fresnel equations by fitting your data to the following model:

$$I_r(\theta) = I_i \times R(\theta) + C$$

Plot a best fit model overlaid with a plot of your data, with error bars. Plot your residuals with error bars. Report the values and uncertainties of all fitted parameters, discuss the meaning (or arbitrariness) of the values of each parameter with respect to verifying/contradicting the theory under test. Plot the fit residuals and discuss them. Report the chi-squared per degree of freedom statistic obtained by your fit, and use it to discuss the likelihood that, if the Fresnel equations were true, you could expect to see data such as yours. Explain any discrepancy; note – saying “we must have made an error....” is not an explanation.

Now endeavor to directly establish the values of $V_0$, $\theta_0$, and $C$ through auxiliary measurements (of your own devising). Fit your previous data to a model in which these parameters are no longer treated as free parameters of the fit, but rather are “hardwired” constants. Plot the new best fit model (overlaid with your data and the previous model). Report the values and uncertainties of all fitted parameters, discuss the meaning (or arbitrariness) of the values of each parameter with respect to verifying/contradicting the theory under test. Plot the fit residuals and discuss them. Report the chi-squared per degree of freedom statistic obtained by your fit, and use it to discuss the likelihood that, if the Fresnel equations were true, you could expect to see data such as yours.

VII. OPTICAL ACTIVITY OF SUCROSE SOLUTION

Many substances exhibit “optical activity,” meaning they rotate the polarization of transmitted light. Sucrose dissolved in water is such a substance. Linearly polarized light passing through $l$ (in units of decimeters) of a sucrose solution of concentration $c = m_{\text{sucrose}}/V_{\text{solution}}$ (in units of grams of sugar /100 ml of solution) will be rotated through an angle:

$$\alpha = \left[ \alpha \right]lc$$

(2)
where \( \alpha \) is constant called the “specific rotation” of the solution. The specific rotation depends strongly on the wavelength of the light, so that it is typically further specified:

\[
\alpha^T_\lambda
\]

Because of a weak temperature dependence, \( T = 20 \) C should also be specified, and \( \lambda_{\text{HeNe}} = 632.8 \) nm for your lasers. Under these conditions, the specific rotation of your sucrose in water solution is \( \alpha^T_{20632} = 57.2144 \text{ deg dm g/100ml} \).

You will use polarimetry to measure the unknown concentration of a water+sucrose solution. There are 8 beakers of sucrose solution that have been pre-prepared. Each group will experiment with a different beaker—retrieve the one with your group’s number on it from the refrigerator in room 208. Your grade in this lab will depend in part on how accurately you determine the true concentration.

**VIII. PROCEDURE**

1. Clean your cuvette thoroughly with distilled water.

2. Fill your cuvette approximately half-full of your groups assigned sucrose solution, using a stirring rod to pour to prevent solution from getting on the clear sides of your cuvette (it is ok to get solution on the frosted sides.)

3. Place the “H” polarizer after mirror M, to definitely set the polarization of the laser to be linear.

4. Adjust the height of the cell so that the beam passes through the top, unfilled part.

5. Place the precision polarizer “P” just after the cell.

6. Position the photodiode to record the light passing through “P”.

7. Turn the room lights off for all measurements in this part of the lab, and take care that no stray light from any group’s desk lamps reaches on the photodiode by placing black aluminum foil around the photodiode.

8. Rotate the “P” polarizer until the voltage is approximately minimized.

9. Tighten the fine adjust lock screw (ask your instructor for help with this), and use the fine-adjust to further minimize the voltage. Now, read off the angle to a precision of 10 arcseconds using the vernier scale on the mount. See the appendix for instructions on how to read the vernier scale.

10. Now raise the cell so that the laser passes complete through the sugar solution. Use the fine-adjust to find a new angle which minimizes the voltage. You will not have to rotate it more than a few degrees!

**IX. ANALYSIS**

Estimate your error in determining both the angle of the polarizer. Calculate your estimate of the sugar concentration, and your uncertainty in this value, using Eq. (2). Note that path length in your cell is 10.0 mm.
X. APPENDIX: READING A ROTATIONAL VERNIER SCALE

The ticks on the coarse scale (the one printed on the rotating bezel) are 2 degrees apart. The ticks on the vernier scale (a.k.a. fine scale, the one on the top of the optic, which doesn’t move as rotate the polarizer) are 10/60ths of a degree apart. The complete reading of the scale is the sum of two readings, acquired as follows. In the picture, the “0” of the vernier scale is slightly to the right of the 30 degrees tick of the coarse scale. The complete angular reading is therefore 30 + \(V\) degrees, where \(V\) is an amount to be determined next. To find \(V\), examine the ticks on the vernier scale lying to the right of the vernier “0”. Identify whichever vernier tick best lines up with ANY tick on the coarse scale. In this picture this happens to be 5th tick to the right of the vernier “0”, and it lines up with the 40 degree tick mark of the coarse scale. We calculate 
\[V = (5\text{th tick to the right of vernier “0”}) \times \left(\frac{10}{60}\text{ degrees per vernier tick}\right) = 50/60\text{ degrees} = 0.83\text{ degrees}.\]
Therefore the complete reading of the scale is 30.83 degrees. Note that it is possible for \(V\) to be larger than 1 degree, but never larger than 2 degrees. Another important note is to IGNORE the “30” and “60” labels on the vernier scale – they are misprinted, and should read “60” and “120”.

You will need to use a 10 cm convex lens as a magnifying glass to make accurate readings of your rotational vernier scale. Beware of parallax – the phenomenon that objects can look aligned or misaligned with one another depending on where you view them from. Here’s a demonstration. Line up your extended thumb with the wall clock. Close your left eye, then your right eye. For one eye the objects will align, for the other eye they will not. In the case of this vernier rotational scale, a tick on the vernier scale is said to line up with a tick on the coarse scale ONLY if it lines up when looking STRAIGHT ALONG the tick. Note that the picture of Fig 5. was taken looking straight down the 5th tick mark. If you looked at this mount from a different angle, some other tick would have appeared to line up, but this would be an incorrect reading.
FIG. 5: This scale reads 30.83 degrees. Do you see it?