Angular Magnification by Optical Instruments
Basic Concepts and Exercises

Note: The discussion and exercises below are from an old Homework (HW) problem set in an earlier intro physics course. In the present course, these exercises are assigned for you to learn and practice the concept of angular magnification. They will not be graded for credit and you do not have to hand in your solutions. You are nevertheless strongly encouraged to take advantage of these exercises: Do them!!

The solutions to these exercises will be posted and/or discussed later.
Review: Angular Magnification by Optical Instruments

As discussed further in part (f) of the following HW problem, P2A.01, the size of the image produced on the eye's retina, $h_e'$, is determined by the angle, $\Theta_e$, subtended by the object being imaged by the eye's lens, namely:

$$|h_e'| \cong \Theta_e d_e'$$

where $d_e'$ is the diameter of the eyeball, which is fixed, i.e., approximately independent of the object the eye is focusing on. We assume here that the angle $\Theta_e$ is measured in radians and that it is very small: $\Theta_e << 1$.

Model of the Vertebrate Eye.

The retinal image size, $|h_e'|$, determines how much detail of the object the eye will actually be able to discern. For sufficiently distant or sufficiently small objects, the object height $h_e$, the object distance $d_e$ from the eye, the angle $\Theta_e$ are related by:

$$\Theta_e \cong h_e / d_e$$

Therefore, $\Theta_e$, and hence object's the retinal image size, $|h_e'|$, becomes very small if:

- either: the object size, $h_e$, is very small, compared to the object distance $d_e$;
- or: the object distance, $d_e$, is very large, compared to the object size $h_e$.

For this reason, the eye is not able to discern any details on a very small object, like a very small insect, even if the object is brought as close as possible to the eye: the closest object distance the eye’s lens can focus on is the near point distance, $d_{near} \approx 20-25cm$. Hence, with $d_e > d_{near}$, the angle $\Theta_e$ can never be larger than $h_e / d_{near}$, and this becomes very small when the object's size, $h_e$, is small.

The magnifying glass and the microscope are optical instruments which allow us to overcome this problem, by producing an enlarged image of the small object. Because of its enlarged size, this image, now serving as the object to the eye, subtends a much larger angle at the eye, and hence produces a much larger image on the retina, than the original small object viewed without magnification. To be viewable by the eye, this enlarged image must be positioned at a distance from the eye which exceeds the near point distance, $d_{near}$.
For the same reason, the eye is also not able to discern any details on a very distant object, even if this object might appear fairly large to us when viewed close-up. For example, if we’re looking at an elephant and a rhinoceros in the zoo, from a close-up distance of, say, 10m, we can clearly tell them apart and discern a great deal of detail of their anatomies: we can see that the elephant has a trunk, but the rhino has a horn on its nose. That’s because the object distance, \(d_e \approx 10\text{m}\), is quite comparable in magnitude to the size (e.g. height) of either animal, which could easily be, say, \(h_e \approx 2\text{m}\) or so. Hence, the angle subtended at the eye by either animal, is about: \(\theta_e \equiv h_e/d_e \approx (2\text{m})/(10\text{m}) = 0.2\) radians. This is not a very small angle and hence the eye produces a sizeable image, of either animal, on our retina.

However, if we observe that same elephant and that same rhino out in the African savanna, from a distance of, say, 10km=10000m, we would hardly be able to tell them apart with the unaided eye: either animal would appear to us as a little grey speck in the landscape, and we certainly wouldn’t be able to tell which one has a trunk, and which one has a horn on its nose. That’s because the object distance, \(d_e \approx 10000\text{m}\), is now much larger in magnitude than the size of either animal, \(h_e \approx 2\text{m}\). Hence the angle subtended at the eye by either animal, is \(\theta_e \equiv h_e/d_e \approx (2\text{m})/(10000\text{m}) = 0.0002\) radians. This is a very small angle and hence the eye produces only a very tiny image, of either animal, on our retina.

The telescope is an optical instrument which allows us to overcome this problem of distinguishing distant elephants from distant rhinos. The telescope accomplishes this by producing a close-up image of a very distant object. Because of its much smaller distance from the eye, this telescope image, now serving as the object to the eye, subtends a much larger angle at the eye, and hence produces a much larger image on the retina, than the original, very distant object viewed without telescope.

As noted above, the retinal image size is proportional to the angle subtended at the eye by the eye’s object. One therefore quantifies the amount of retinal image magnification achieved by an optical instrument in terms of the so-called angular magnification, denoted by \(M_\theta\), The basic definition of \(M_\theta\), applies to both kinds of instruments: magnifying glasses and microscopes which produce enlarged images of small objects; and telescopes which produce close-up images of distant objects. In both cases, \(M_\theta\), is defined to compare the angle subtended at the eye by the image produced from the object with the optical instrument, to the angle subtended by the eye when the same object is viewed without optical instrument. Viewing the original object without optical instrument will be called the “reference configuration”; viewing the object’s image, produced with the optical instrument will be called the “instrument configuration”, in the following.
Reference Configuration

\[ \text{d}_{\text{ref}} = d_{\text{Near}} \quad \text{for microscope, magnifying glass} \]

\[ \text{d}_{\text{ref}} = d_1 = \text{distance orig. object to obj. lens, for telescope} \]

\[ h_{\text{ref}} = h_1 = h = h_0 = \text{orig. object size, for all cases} \]

Instrument Configuration

\[ d_{e} = -d_2' = |d_2'| \quad \text{for microscope, telescope} \]

\[ d_{e} = -d' = |d'| \quad \text{for magnifying glass} \]

\[ h_{e} = |h_2'| \quad \text{for microscope, telescope} \]

\[ h_{e} = h' \quad \text{for magnifying glass} \]
In the instrument configuration, the (final) image produced by the instrument serves as the object to the eye and we will from now on reserve the symbols
\[ h_e, \ d_e, \ \Theta_e \]
to denote the object height, the object distance measured from the eye, and the angle subtended at the eye by this object, respectively. But be clear here: this “object” that the eye “sees” in the instrument configuration is the same as the (final) image produced by the optical instrument! So, when analyzing the inner workings of such an instrument, the eye’s object height, \( h_e \), is the same as the instrument’s final image height, denoted, e.g., by \( h_2’ \) in a telescope or a microscope:
\[ h_e = h_2’. \]
Likewise, the eye’s object distance, \( d_e \), measured from the eye to the instrument’s final image, is closely related to the instrument’s final image distance, denoted, e.g., by \( d_2’ \) in a telescope or a microscope. Here, \( d_2’ \) is measured from the eyepiece lens of the instrument. Provided the eye is positioned very close to the eyepiece, we then have
\[ d_e \equiv -d_2’ = |d_2’| \]
Note here that \( d_2’ \) is negative here, since the eye is positioned on the outgoing side of the eyepiece lens, and the image must be produced “in front of” eye, i.e., on the opposite (not outgoing) side of the eyepiece. So the final image produced by the instrument is always a virtual image to the eyepiece (\( d_2’<0 \)), and at the same time it is a real object (\( d_e>0 \)) to the eye.

The foregoing relations between \( h_e, \ h_2’, \ d_e \) and \( d_2’ \) are illustrated in the figures and discussion in the Microscope document posted on the “Examples” page, [http://www.physast.uga.edu/classes/phys1212/schuttler/examples/Lnotes/Microscope110205Text+Figs.pdf](http://www.physast.uga.edu/classes/phys1212/schuttler/examples/Lnotes/Microscope110205Text+Figs.pdf).

You should download this document and carefully study especially the figures for Steps 11-14 shown therein. Then read the foregoing and following paragraph again. Both reference and instrument configuration are also illustrated in the figure for HW problem P2A.01 below: look it over!

In the reference configuration, the original object, without magnification, serves as the object to the eye. To clearly distinguish the object parameters (height, distance and angle subtended) in the reference configuration from those describing the eye’s object in the instrument configuration, and we will from now on reserve the symbols
\[ h_{ref}, \ d_{ref}, \ \Theta_{ref} \]
to denote the object height, the object distance measured from the eye, and the angle subtended at the eye by the original object, when viewed in the reference configuration. (And again, the symbols \( h_e, \ d_e, \ \Theta_e \) are from now on reserved for the eye’s object in the instrument configuration.)

As noted above, from \( h_e, \ d_e \) and \( h_{ref}, \ d_{ref} \), we get:
\[ \Theta_e \equiv h_e / d_e \quad \text{and likewise} \quad \Theta_{ref} \equiv h_{ref} / d_{ref} \]
Given the two angles, $\Theta_e$ and $\Theta_{\text{ref}}$, subtended by the eye’s object in instrument and reference configuration, respectively, the angular magnification is defined by the ratio:

$$M_\Theta = \frac{\Theta_e}{\Theta_{\text{ref}}}.$$  

The foregoing general definition of $M_\Theta$ applies to both types of “magnifying” optical instruments: magnifying glasses and microscopes, as well as telescopes. However, the choice of the reference configuration for the two types is different, for obvious reasons:

**Microscopes and magnifying glasses** enlarge small objects that can be arbitrarily positioned at any desired distance from the eye, if no instrument is available. Hence, one chooses as the reference configuration the closest possible distance of the object at which the eye can still focus on the object, the near point distance, and therefore one uses

$$d_{\text{ref}} = d_{\text{near}}$$  

as the reference object distance for microscopes and magnifying glasses.

**Telescopes** on the other hand produce close-up images of very distant objects. And the distance of these objects when viewed by the eye without instrument can not be easily changed. Hence one assumes that the distant original object is positioned at the same (distant) location for both the reference and the instrument configuration. The (very large) distance of the original object from the eye in the reference configuration is then essentially the same as the distance of that same original object from the telescope in the instrument configuration. Since the length of the telescope, $L$, is negligible compared to $d_{\text{ref}}$, one sets

$$d_{\text{ref}} = d_1$$  

where $d_1$ is the distance of the original object, measured from the objective lens (lens 1) of the telescope. One then also takes the limit

$$d_1 \rightarrow \infty, \text{ hence: } 1/d_1 \rightarrow 0$$  

since $d_1$ is typically much, much larger than the focal length, $f_1$, of the objective lens.

The object height in the reference configuration, $h_{\text{ref}}$, is obviously just the size of the original object, denoted here by $h_0$, and this is the same for microscopes, magnifying glasses and telescopes:

$$h_{\text{ref}} = h_1 = h_0.$$  

Since the same original object, of size $h_0$, also serves as the object for the instrument’s objective lens, the object height for the objective lens, $h_1$, is also the same as $h_0$.

From the foregoing general definition of the angular magnification, and the specifications of the reference configurations, one can then derive various exact or approximate formulae to calculate $M_\Theta$ directly from the focal length(s) of the lenses (or mirrors!) and various other distances in the instrument. The derivation of one such equation is assigned as part (h) in HW problem P2A.02. We will not discuss these formulae here in any detail. If needed in any HW or exam problem, they will be provided in the problem statement, as e.g., in P2A.03.
P2A.01  Angular and Retinal Image Magnification

An elephant is viewed from a large distance in two different viewing configurations:

(a) without any optical instrument, at the (very large) object distance, \( d_{\text{ref}} \), measured from the eye. We call this the “reference configuration”, as shown in Fig. (a) below. The height of the object as seen by the eye, denoted by \( h_{\text{ref}} \) in this configuration, is just the original height of the elephant. The angle subtended by this object at the eye is denoted by \( \Theta_{\text{ref}} \).

(b) through a magnifying optical instrument: a telescope. We call this the “instrument configuration”, as shown below in Fig.(b). The eye in this case does not “see” the original elephant at its actual large distance, \( d_{\text{ref}} \), and height, \( h_{\text{ref}} \), but rather, it sees the final, virtual image \( \text{Img2} \) produced by the instrument’s eyepiece (lens 2) at the final image distance, \( d_2' < 0 \), measured from the eyepiece, with image height \( h'_2 \). Thus, the instrument’s final virtual image serves as a real object to the eye, with object distance \( d_e \), measured from the eye, and object height \( h_e \), given by

\[
d_e \equiv -d_2' = |d_2'| \quad \text{and} \quad h_e = h_2',
\]

assuming, re \( d_e \), that the distance between eye and eyepiece is negligible. The angle subtended at the eye by this object (=final image by instrument, \( \text{Img2} \)) is denoted by \( \Theta_e \).

Note also that in the instrument configuration, the original elephant serves as the object \( \text{Obj1} \) to the objective lens (lens1) of the telescope. Hence, in the instrument configuration, \( d_{\text{ref}} \) and \( h_{\text{ref}} \) are the object distance and object height for the telescope’s objective lens:

\[
d_1 \equiv d_{\text{ref}} \quad \text{and} \quad h_1 = h_{\text{ref}},
\]

assuming, re \( d_1 \), that the length of the telescope, \( L = d_1' + d_2 \), is negligible compared its distance \( d_{\text{ref}} \) from the elephant.

In the following problem, you do not have to analyze what goes on inside of the telescope. Rather, the object distance, \( d_e \), and height, \( h_e \), of the telescope’s final image \( \text{Img2} \) from the eye are given, for the instrument configuration, see Fig(b). Also, \( d_{\text{ref}} \) and \( h_{\text{ref}} \) for the reference configuration are given, see Fig(a). Rather, the focus of the following problem is on the images formed on the retina of the eye, in the foregoing two configurations.

Assume the (fixed) diameter of the eyeball is \( d_e = 4.5 \text{cm} \), with all other inputs \( d_{\text{ref}}, h_{\text{ref}}, d_e, h_e \) given in the figures below.

(a) For the reference configuration, Fig. (a), find \( \Theta_{\text{ref}} \) and the height of the elephant’s image on the retina, denoted by \( h_{\text{ref}}' \). Use \( \Theta_{\text{ref}} \equiv \tan(\Theta_{\text{ref}}) \) and explain why this is allowed!

(b) For the instrument configuration, Fig. (b), find \( \Theta_e \) and the height of the elephant’s image on the retina, denoted by \( h_e' \). Use \( \Theta_e \equiv \tan(\Theta_e) \) and explain why this is allowed!
(c) Compare the **retinal image heights** obtained with and without optical instrument: calculate the “**retinal magnification**”, defined by

\[ M_{\text{retina}} = \frac{|h_e'|}{|h_{\text{ref}}'|} . \]

(d) Compare the **angles subtended by the eye's objects at the eye, as seen with and without optical instrument**: calculate the “**angular magnification**”, defined by

\[ M_{\theta} = \frac{\Theta_e}{\Theta_{\text{ref}}} . \]

(e) Compare \( M_{\text{retina}} \) and \( M_{\theta} \). Do they agree?

(f) Use the definitions of \( \Theta_{\text{ref}} \) and \( \Theta_e \), the approximations \( \Theta_{\text{ref}} \cong \tan(\Theta_{\text{ref}}) \) and \( \Theta_e \cong \tan(\Theta_e) \), and the lateral magnification equation for the eye’s object and retinal image, applied to both the reference and the instrument configuration, to prove that

\[ |h_{\text{ref}}'| = \Theta_{\text{ref}} d_e' \quad \text{and} \quad |h_e'| = \Theta_e d_e'. \]

Then use that result to prove that

\[ M_{\text{retina}} = M_{\theta} . \]

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Fig. (a) Reference Configuration; and Fig. (b) Instrument Configuration of a Telescope
P2A.02 Angular Magnification by Magnifying Glass

A magnifying glass (MG) is a convergent lens, with a short focal length, \( f > 0 \), and \( f \) less than the eye’s near point distance, \( d_{\text{Near}} \). From a small object, the MG produces an image, seen by the eye at a distance \( d_e > 0 \), measured from the eye. The original small object is located within the focal length from the lens, i.e., at a distance less than \( f \). By adjusting this object distance, the distance \( d_e \) can be chosen anywhere between the eye’s near point distance, \( d_{\text{Near}} \), and very far away (\( \infty \)). The eye is positioned very close to the MG; so the distance from eye to MG can be neglected.

Suppose a small insect, of size 1.2mm, is placed to the left of a MG with focal length +4.0cm; and the insect’s MG image is viewed by the eye, positioned to the right of the MG. The insect’s position is adjusted so that its MG image appears at distance \( d_e \) of about, or somewhat greater than, \( d_{\text{Near}}=20\text{cm} \).

(a) Draw a clean, big (full-page !) figure, showing a complete ray diagram for the formation of the insect’s image from the original object, with the object and image shown as vertical arrows, perpendicular to the optical axis. In addition to the ray diagram, the figure should indicate the object and image distances, \( d \) and \( d' \), drawn as horizontal arrows from lens to object and lens to image, resp., parallel to the optical axis. The figure should also show the positioning of the eye, relative to the MG.

All elements of the drawing should be properly labeled: optical axis, lens, \( F, F', P, P' \)-rays, focal points \( F \) ad \( F' \), object and image, distances \( d \) and \( d' \), as well as object and image heights, \( h \) and \( h' \), resp. The labeling should also clearly indicate the “incoming” and the “outgoing” side of the lens.

The drawing does not have to be to scale, but the relative positioning of all elements along the optical axis must be correct. A “clean” drawing requires a ruler (straightedge)!

(b) Assume now that the MG image is formed at the eye’s near point. State or calculate: the distances \( d \) and \( d' \), and the heights, \( h \) and \( h' \).

Hint: \( d_e = |d'| \). Explain why!

(c) From the results in (b), calculate the angle, \( \Theta_e \), subtended at the eye by the insect’s MG image, as seen by the eye. Also indicate this angle (schematically) in the drawing you made in part (a), using a pen(cil) of a different color.

Hint: \( \Theta_e \approx \tan(\Theta_e) \). Explain why!

(d) Make a drawing of the “reference configuration”: the original insect being viewed by the eye without microscope. This should show a horizontal optical axis, the eye, and the original insect as a vertical arrow of height \( h_{\text{ref}} \), positioned at the distance \( d_{\text{ref}} \) from the eye, with \( d_{\text{ref}} \) shown as a horizontal arrow from eye to insect, parallel to the optical axis. Also indicate the angle \( \Theta_{\text{ref}} \), subtended at the eye by the insect.
All elements of the drawing should be properly labeled: optical axis, eye, object (insect), height $h_{ref}$, distance $d_{ref}$ and angle $\Theta_{ref}$. Again: the drawing does not have to be to scale.

(e) Referring to the figure from part (d), calculate the angle $\Theta_{ref}$ subtended at the eye by the insect when viewed at the reference distance, $d_{ref}$, without microscope. Hints: $h_{ref}=h$ and $d_{ref}=d_{near}$ and $\Theta_{ref}\approx \tan(\Theta_{ref})$. Explain why!

(f) Find the angular magnification, $M_\theta$, achieved with the parameters given in (b), using the angles $\Theta_e$ and $\Theta_{ref}$ found in parts (c) and (e).

(g) Repeat parts (b), (c), (e) and (f) if the object (insect) is positioned so that it's MG image appears at a very large distance from the eye, i.e., assuming $d_{e} >> d_{near}$ and hence $d_{e} >> f$. Hint: you can take $d_{e} \rightarrow \infty$ here, and hence set $1/d_{e}=0$. Explain why!

(h) Use the image formation eqs. to prove in general that, for any choice of MG image position $d_{e}$, the MG angular magnification is given by

\[ M_\theta = d_{Near} \left( \frac{1}{f} + \frac{1}{d_{e}} \right) \]

Hint: Go through the same calculation steps, and use the same Hints, as in parts (b), (c), (e) and (f). Use the image formation eqs to express $h'$ in terms of $h$, $d$ and $d_{e}$, and to express $d$ in terms of $f$ and $d_{e}$. Then express both $\Theta_e$ and $\Theta_{ref}$ in terms of $h$, $d_{Near}$, $f$ and $d_{e}$. Do all this without plugging in any numbers.
Angular Magnification by Microscope

A microscope is built with an objective lens of focal length \( f_1 = 3.0 \text{cm} \) and an eyepiece lens of \( f_2 = 4.0 \text{cm} \). A small object of 0.1mm diameter, is placed 3.3cm in front of the objective lens. You want the final image, produced by the eyepiece, to appear 40cm from the eye. The eye is positioned very close to the eyepiece; so the distance from eye to eyepiece can be neglected. See below for Hints!

(a) State or calculate, and then tabulate, the object and image distances and heights of both lenses:
\[
\begin{align*}
d_1, & \quad d_1', \quad d_2, \quad d_2', \quad h_1, \quad h_1', \quad h_2, \quad h_2'.
\end{align*}
\]

(b) Find the distance, \( L \), between the two lenses.

(c) Find the angle, \( \Theta_e \), subtended at the eye by the final image produced by the eyepiece.

(d) Calculate the reference angle, \( \Theta_{\text{ref}} \), subtented at the eye when viewing the same small object at closest possible distance \( d_{\text{ref}} = d_{\text{Near}} = 20 \text{cm} \), without microscope.

(e) From \( \Theta_e \) and \( \Theta_{\text{ref}} \), calculate the angular magnification, \( M_\theta \), achieved with the microscope, when compared to viewing the same small object at the near point distance without microscope.

(f) Calculate \( M_\theta \) from the general angular magnification eq.
\[
M_\theta = M_{\text{eyepiece}} \left| m_1 \right|
\]

Here \( M_{\text{eyepiece}} \) is the angular magnification by the eyepiece, acting as a magnifying glass, and \( m_1 \) is the objective lens lateral magnification.

Hints for parts (a-d):

(1) Carefully study and refer to the figures shown in
http://www.physast.uga.edu/classes/phys1212/schuttler/examples/lnotes/Microscope110205Text+Figs.pdf

(2) \( d_e = |d_2'| \) Why? Drawing !

(3) \( h_e = |h_2'| \) Why? Drawing !

(3) \( d_{\text{ref}} = d_{\text{Near}} \) Why? Drawing !

(4) \( h_{\text{ref}} = |h_1| \) Why? Drawing !

Hints for part (f):

(5) Use, without proof, the result from P2A.02 part (h), Eq.(M), to calculate \( M_{\text{eyepiece}} \) with \( f_2 \) being the focal length of the eyepiece.
P2A.04  Angular Magnification with a Divergent Lens

Download:

http://www.physast.uga.edu/classes/phys1212/schuttler/examples/lnotes/Microscope110205Text+Figs.pdf

On Page 4 of that document, you will find three additional Exercises. Solve the problem stated in

Exercise #2

This is the same problem set-up as the example on the preceding pages of the Microscope document, except that the convergent lens 2 is replaced by a divergent lens with focal length

\[ f_2 = -2.50\text{cm}. \]
P2A.05 Telescopic Camera with a Divergent Projection Lens.

The “old” Telescopic Camera problem shown on the page below is taken from the “STP Collection” link, posted on the course website under “Homework”, on the page http://www.physast.uga.edu/classes/phys1212/schuttler/hw/pt02_ImageFmOptInst.html. The solution for this “old” problem is also posted on that page. (Read below for Notation and a small Typo Correction!). Study that “old” problem before working on the HW problem assigned here. Then solve the following as your HW problem P2A.05:

Modify the “old” Telescopic Camera described below by replacing the projection lens (Lens 2) with a divergent lens of negative focal length,

\[ f_2 = -1.0\text{cm}. \]

Otherwise, use all the same input parameters as given in the “old” problem below, answer the same questions as stated in Parts (a), (b) and (c) in the “old” problem, and in addition do this:

(d) Draw a clean, complete, fully labeled, big, full-page ray diagram for the formation of Image 2 from Object 2 by Lens 2 (with \(f_2<0\)). It doesn't have to be drawn to scale, but the relative positioning of lens, image, object and focal points, \(F_2\) and \(F_2'\); and the orientations of object and image, must be shown correctly.

Hints:
(1) Keep at least 4 significant digits in all calculations, to ensure accurate results.
(2) In part (a), the term “intermediate image” means: “Image 1, produced by Lens 1”.
(3) In Part (c), you first need to calculate the relevant angles, \(\Theta_e\) and \(\Theta_{ref}\), for the “Instrument Configuration” and for the “Reference Configuration”, resp. Then calculate the angular magnification, \(M_o\), from those two angles. The “Instrument Configuration” here consists of the eye viewing the giraffe’s photographic image, from the 25cm (=near point) distance. In the “Reference Configuration”, the eye is viewing the original giraffe itself, without use of any instrument, but from the same 200m distance at which the photograph was taken.

Notation:
In the solution to the “old” problem, posted on the page given above, the following notation is used (see also the “Read Me First” link posted on that page):

\[
\begin{align*}
\theta', \Theta & \quad \text{instead of} \quad \theta, \Theta \\
\mathbf{d}_1, \mathbf{d}_1', \mathbf{d}_2, \mathbf{d}_2', \mathbf{h}_1, \mathbf{h}_1', \mathbf{h}_2, \mathbf{h}_2', \mathbf{L} & \quad \text{instead of} \quad \mathbf{d}_1, \mathbf{d}_1', \mathbf{d}_2, \mathbf{d}_2', \mathbf{h}_1, \mathbf{h}_1', \mathbf{h}_2, \mathbf{h}_2', \mathbf{L}
\end{align*}
\]

Typo Correction:
The posted solution to the “old” problem states, incorrectly: “Image on film is 3.32cm tall”. The correct statement is: “Image on film is 3.15cm tall”. The correct image height, 3.15 cm, was used in the subsequent solution for Part (c).
3. A "telescopic lens" is used in a photographic camera to project an image of a very distant object onto a photosensitive film inside the camera. Suppose the "telescopic lens" consists of two convergent lenses, 1 and 2, with focal lengths $f_1 = 20.0\text{ cm}$ and $f_2 = 1.0\text{ cm}$, respectively, and the film is mounted 10.0 cm to the right of lens 2.

Hint: $1\text{ m} = 100\text{ cm}$.

\[ f_1 = 20.0\text{ cm} \quad f_2 = 1.0\text{ cm} \quad \text{film} \]

(a) How far from lens 2 must lens 1 be mounted in order to obtain a sharp image on the film?

Hints: (i) Find first the position of the intermediate image relative to lens 2.

(ii) \( \frac{1}{f_1} - \frac{1}{D} \approx \frac{1}{f_1} \) if \( D \gg f_1 \).

(b) Using this "telescopic lens", you take a photograph of a giraffe, 3.50 m tall, from a distance of 200 m (\( \gg f_1 \)). How tall is the giraffe's image on the film?

(c) Suppose the film with the giraffe from (b) is developed without any further magnification and you look at the photograph from a distance of 25 cm. What "angular magnification" have you achieved now, compared to viewing the original giraffe directly (with your bare eyes) from a distance of 200 m.