ASTR 1110H – Solutions to Quantitative Exercises

Chapter 1

#1. Use Kepler’s 3rd law (as shown at the top of page 25): \(D^3 = A \cdot P^2\)

The asteroid has a period of revolution, \(P\), of 8 years, so, since \(A = 1 \text{ AU}^3/\text{yr}^2\), \(D\) equals the cubic root of \(1 \text{ AU}^3/\text{yr}^2 \cdot 8^2 = 4 \text{ AU}\).

#2. Again referring to Kepler’s 3rd law, if the distance, \(D\), is cut in half, then \(D^3\) will be \(1/8^{th}\) as large. So \(P^2\) will also be \(1/8^{th}\) as large. Thus, \(P\) will be \(1/8 = 0.354\) times as large. Since Mercury’s period is 88 days (page 182), the period of this hypothetical planet would be \(0.354 \times 88 = 31\) days.

Pluto’s semi-major axis is 39.44 AU (Table 1.1 on page 24). So the distance, \(D\), for this hypothetical planet would be \(2 \cdot 39.44 = 79\) AU. So \(D^3 = 493,039 = P^2\). Thus, \(P = \sqrt{493,039} = 702\) years.

#3. Now use the 2nd equation on page 25, Newton’s version of Kepler’s 3rd law: \(D^3 = A \cdot (M_1 + M_2) \cdot P^2\). In this case, \(M_1\) represents the mass of the Sun and \(M_2\) represents the mass of the Earth. But since the mass of the Sun is 333,000 times larger than the mass of the Earth (Table 2.1 on page 31), we can neglect the mass of the Earth.

The first part of the question says to keep the mass of the Sun the same but to change its diameter. Since diameter doesn’t show up in Newton’s version of Kepler’s 3rd law, changing the diameter will have no effect on the period of the Earth. The second part says to keep the diameter the same, but double the mass of the Sun (keeping \(D = 1 \text{ AU}\) the same). So, using a little bit of algebra, we get \(P^2 = D^3 / (A \cdot M_1) = 1 / (1 \cdot 2) = 1/2\). So \(P = \sqrt{1/2} = 0.71\) years.

#4. For this problem, we can again use Newton’s version of Kepler’s 3rd law. But this time, \(M_1\) is the mass of the Earth (and \(M_2\), the mass of the satellite, is negligible). And we must convert the Earth’s mass into solar mass units, i.e., the value of \(M_1\) is 1/333,000. Likewise, the distance of the satellite 10,000 km above the Earth’s surface is 16,378 km (Earth’s radius is 6,378 km; see inside front cover of textbook) from the Earth’s center, but we must convert this into AU. The value of 1 AU = 150,000,000 km (page 14), so 16,378 km/150,000,000 km/AU = 0.00011 AU – this is the value of \(D\).

Now we’re all set to find \(P\): \(P^2 = D^3 / (A \cdot M_1) = (0.00011)^3 / (1 \cdot 1/333,000) = 4.43 \times 10^{-7}\), so \(P = 4.43 \times 10^{-7} = 6.66 \times 10^{-4}\) years = 0.243 days = 5.83 hours.

#5. We can go back to Kepler’s original version of his 3rd law for this problem, since the spacecraft orbits the Sun (and so \(M\), will simply equal 1). For the trip to go to Saturn, the time it takes will be \(\text{half}\) of the orbital period, \(P\) (which equals \(5^3 = 11.18\) years); so, about 5.6 years. For the trip to Mars, we need to know the distance to Mars (1.5 AU
from the inside front cover of your textbook) and the semi-major axis of the orbit, D, which would be (the distance to Mars plus the distance to Earth)/2 = (1.5 + 1)/2 = 1.25 AU. Then P = $1.25^3 = 1.40$ years.