Solutions to Conceptual Practice Problems
PHYS 1112 In-Class Exam #1A+1B
Thu. Feb. 4, 2010, 9:30am-10:45am and 11:00am-12:15pm

CP 1.01: A student runs westward at 3m/s, away from a vertical plane mirror, while the mirror, mounted on wheels, travels eastward at 7 m/s (with both speeds given relative to the ground). The speed at which the student’s image moves and its direction, relative to the ground, is

(A) 4m/s westward
(B) 4m/s eastward
(C) 10m/s eastward
(D) 17m/s eastward
(E) 17m/s westward

Answer: (D)

This is the same problem as HW P02.04, except that the direction of motion of the runner, of the mirror and hence the image has been reversed: both runner and image are moving away from the mirror instead of towards it. The speed of runner relative to mirror is \((3 + 7)\)m/s = 10m/s in westward direction. Hence, the image’s speed is also 10m/s, but in eastward direction, relative to the mirror. Since the mirror also moves eastward at 7m/s relative to the ground, the speed of mirror relative to ground and of image relative to mirror add up, to \((10 + 7)\)m/s = 17m/s eastward.

CP 1.02: Sound waves (including ultrasound) have a speed of wave propagation \(v_{\text{Air}} = 346\)m/s in air and \(v_{\text{Water}} = 1497\)m/s in water. Also, note that \(\sin(13.364^\circ) = 346/1497\). A narrow ultrasound beam striking the flat water surface of your swimming pool

(A) will have an angle of refraction smaller than the angle of incidence if the beam is incident from above the water surface;
(B) will have an angle of refraction greater than the angle of incidence if the beam is incident from below the water surface;
(C) cannot undergo total internal reflection if incident from above the water surface, regardless of the angle of incidence;
(D) will undergo total internal reflection if incident from above the water surface with an angle of incidence of 30°
(E) will undergo total internal reflection if incident from below the water surface with an angle of incidence of 30°.
Answer: (D)

To fully understand the following discussion of the Answer, it is imperative that you make a drawing of the air-water interface, of the normal to that interface, and of the incident and refracted rays, for each case discussed (incidence from above and incidence from below), with correct arrows attached to the rays (indicating their direction of propagation), and with correct relative angular size relations shown (Should angle of incidence be drawn as greater than, or should it be drawn as less than, angle of refraction?).

Refraction of sound at the air-water interface differs from refraction of light at the same interface in that light travels slower in water than in air, whereas sound travels faster in water than in air.

Total internal reflection (TIR) occurs if (a) the sound beam is incident from the low-speed medium (=air, for sound) and gets refracted into the high-speed medium (=water, for sound) and (b) the angle of incidence $\Theta_1$ exceeds the critical angle, defined by $\sin(\Theta_{(\text{crit})}) = v_1/v_2$, with $v_1 < v_2$. Hence, the sound beam can undergo TIR only if incident from air into water, but never if incident from water into air! Also, for incidence from air into water, $\sin(\Theta_{\text{(crit) Air}}) = 346/1497$, i.e., $\Theta_{\text{Air}}^{(\text{crit})} = 13.364^\circ$ from the information given.

The foregoing two TIR prerequisites (a) and (b) are satisfied by the conditions stated in (D), but not by the conditions stated in (E). For this reason, (D) is correct and both (E) and (C) are wrong.

Regardless of whether the sound beam is incident from air into water or from water into air, by Snell’s law

$$\sin(\Theta_{\text{Water}}) = \left(\frac{v_{\text{Water}}}{v_{\text{Air}}}\right) \sin(\Theta_{\text{Air}})$$

where $\Theta_{\text{Air}}$ and $\Theta_{\text{Water}}$ are the angles of the sound beam in air and water, respectively, measured from the normal, and $v_{\text{Air}} = 346\text{m/s}$ and $v_{\text{Water}} = 1497\text{m/s}$ are the corresponding speeds. Since $v_{\text{Water}} > v_{\text{Air}}$ this implies that $\sin(\Theta_{\text{Water}}) > \sin(\Theta_{\text{Air}})$ and hence

$$\Theta_{\text{Water}} > \Theta_{\text{Air}}$$

That is, as a matter of general principle, the beam always has the greater angle to the normal when traveling in the medium with the greater speed. (And this is always true for any kind of wave being refracted between any two different media!)

So, if incident from air, the sound beam is refracted away from the normal upon entering the water; and if incident from water, the sound beam is refracted towards the normal upon entering the air (provided that refraction occurs at all, i.e., absent TIR). For these reasons answers (A) and (B) are wrong.

CP 1.03: If a real object is placed between the focal point and the lens for a convergent lens $(f > 0)$, then the image is

(A) virtual, erect and enlarged in height relative to the object
(B) virtual, erect and reduced in height relative to the object
(C) real, inverted and reduced in height relative to the object
(D) real, inverted and enlarged in height relative to the object
(E) real, erect and reduced in height relative to the object

Answer: (A)

For a real object \( d > 0 \). Also, "placed between focal point and lens" means:

\[ f > d > 0 . \]

Therefore, by taking the inverse of the inequality \( f > d \) and using \( d > 0 \), it follows

\[ 0 < 1/f < 1/d \]

and hence

\[ -1/d' = 1/d - 1/f > 0 \quad \text{Eq.}(3.1) \]

From this it follows that \( d' < 0 \), i.e., the image is virtual; and also \( m = -d'/d > 0 \), i.e. the image is erect.

However, from Eq.(3.1), it also follows that

\[ -1/d' = 1/d - 1/f < 1/d \quad \text{Eq.}(3.2), \]

since \( 1/f > 0 \). Eq.(3.2) and \(-d' > 0 \) [from Eq.(3.1)], then implies \(-d' > d > 0 \), and hence

\[ m = (-d')/d > 1 , \]

i.e. the image is enlarged. So, the answer is (A).

Lastly, if you do not like general mathematical proofs, like the one above, you could also answer this question simply by plugging in some numbers for \( d \) and \( f \). Take, e.g, \( f = 4 \text{cm} \)

and \( d = 3 \text{cm} \) so that \( 0 < d < f \), i.e., the object is placed between focal point and lens. Then, you’ll get \( d' = (1/f - 1/d)^{-1} = [(1/4) - (1/3)]^{-1} \text{cm} = -12 \text{cm} \) and from that \( m = -d'/d = -(-12)/3 = +4 \). Therefore:

Since \( d' < 0 \), the image is virtual.

Since \( m > 0 \), the image is erect, relative to object

Since \(|m| > 1\), the image is enlarged in absolute height, relative to object.

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**CP 1.04:** A thin lens and a curved mirror both have the same focal length when surrounded by air (with index of refraction \( n_{\text{Air}} = 1 \)). If the lens and the mirror are submerged in water (with index of refraction \( n_{\text{Water}} > 1 \)) what happens to the focal length \( f_L \) of the lens and the focal length \( f_M \) of the mirror?

(A) \( f_L \) changes and \( f_M \) stays the same;
(B) \( f_L \) stays the same and \( f_M \) changes;
(C) \( f_L \) increases and \( f_M \) increases;
(D) \( f_L \) decreases and \( f_M \) decreases;
(E) \( f_L \) stays the same and \( f_M \) stays the same.

Answer: (A)

The lens forms images by bending light rays by means of refraction. For each light ray involved in forming the image, refraction occurs at the front and at the rear surface of the lens; both of these surfaces are interfaces between the lens material (e.g., glass) and the surrounding medium. By Snell’s law, the angles of refraction at both surfaces depend on both the index of refraction (IoR) of the lens material and on the IoR of the surrounding medium (air or water). Therefore, for a fixed object position \( d \), the image position \( d' \) depends on (i.e., changes with) the IoR of the surrounding medium; and therefore the focal length \( f_L \equiv (1/d + 1/d')^{-1} \) changes with the IoR of the surrounding medium: \( f_L \) changes when the lens is submerged from air into water.

The mirror forms images by bending light rays by means of reflection. For each light ray involved in forming the image, reflection occurs at the curved surface of the mirror. By Archimedes’ law \( (\bar{\Theta} = \Theta) \), the angle of reflection \( (\bar{\Theta}) \) for each light ray at that surface depends only on the angle of incidence \( (\Theta) \), but it does not depend on the surrounding medium (air or water). Therefore, for a fixed object position \( d \), the image position \( d' \) does not depend on (i.e., does not change with) the surrounding medium; and therefore the focal length \( f_M \equiv (1/d + 1/d')^{-1} \) does not change when we change the surrounding medium: \( f_M \) is the same regardless of whether the mirror is submerged in air or water.

CP 1.05: An observer \( O \), facing a mirror, observes a light source \( S \), with the observer, source and mirror positioned as shown here:

\[
\begin{array}{c}
O \\
\text{mirror} \\
\text{1} \rightarrow \text{3}
\end{array}
\begin{array}{c}
S \\
\text{4}
\end{array}
\]

Where does \( O \) perceive the mirror image of \( S \) to be located?

(A) Position 1.
(B) Position 2.
(C) Position 3.
(D) Position 4.
(E) The image of S cannot be seen by O in the configuration shown above.

Answer: (D)

Any rays emerging from S which strike the mirror are reflected off the mirror (acc. to Archimedes’ law) in such a way that, after reflection, they appear to be emerging from Position 4. This is true (a) for all rays emerging from S regardless of where the rays from S strike the mirror; and (b) regardless of how far up or down the mirror extends, i.e., even if the mirror surface does not cover the wall area immediately opposite of S.

It is critically important that you draw your own big, clean picture of this set-up, tracing a few (at least two) rays from S to the mirror and then, after reflection with \( \bar{\Theta} = \Theta \), tracing the reflected rays backwards to see where they intersect behind the mirror: at Position 4.

This is similar to the in-class quiz on buying the “Shortest Possible Mirror on the Wall”. You can still see your own feet (on the ground) in this mirror, even though the mirror extends downward to only half of your height above the ground.

CP 1.06: Sound waves (including ultrasound) have a speed of wave propagation \( v_{\text{Air}} = 346 \text{m/s} \) in air and \( v_{\text{Water}} = 1497 \text{m/s} \) in water. Also, note that \( \sin(13.364^\circ) = \frac{346}{1497} \).

A narrow ultrasound beam striking the flat water surface of your swimming pool

(A) will undergo total internal reflection if incident from below the water surface for any angle of incidence greater than 13.364\(^\circ\);
(B) will undergo total internal reflection if incident from above the water surface with an angle of incidence of 8.5\(^\circ\);
(C) will undergo total internal reflection if incident from below the water surface with an angle of incidence of 8.5\(^\circ\);
(D) will have an angle of refraction smaller than the angle of incidence if the beam is incident from below the water surface;
(E) will have an angle of refraction greater than the angle of incidence if the beam is incident from below the water surface.

Answer: (D)

To fully understand the following discussion of the Answer, it is imperative that you make a drawing of the air-water interface, of the normal to that interface, and of the incident and refracted rays, for each case discussed (incidence from above and incidence from below), with correct arrows attached to the rays (indicating their direction of propagation), and with correct relative angular size relations shown (Should angle of incidence be drawn as greater than, or should it be drawn as less than, angle of refraction ?).
Refraction of sound at the air-water interface differs from refraction of light at the same interface in that light travels slower in water than in air, whereas sound travels faster in water than in air.

Total internal reflection (TIR) occurs if (a) the sound beam is incident from the low-speed medium (=air, for sound) and gets refracted into the high-speed medium (=water, for sound) and (b) the angle of incidence $\Theta_1$ exceeds the critical angle, defined by $\sin(\Theta^{\text{crit}}_1) = v_1/v_2$, with $v_1 < v_2$. Hence, the sound beam can undergo TIR only if incident from air into water, but never if incident from water into air! Also, for incidence from air into water, $\sin(\Theta^{\text{crit}}_{\text{Air}}) = 346/1497$, i.e., $\Theta^{\text{crit}}_{\text{Air}} = 13.364^\circ$ from the information given.

The foregoing two TIR prerequisites (a) and (b) are not both satisfied by the conditions stated in (A), (B) or (C). For this reason, (A), (B) and (C) are wrong. [In (A) and (C) incidence is from below, i.e. from water into air, hence TIR cannot occur, regardless of angle of incidence. In (B), incidence is from above, i.e. from air into water, as required for TIR condition (a); but the angle of incidence, $\Theta = 8.5^\circ$, is less than the critical angle $\Theta^{\text{crit}}_{\text{Air}} = 13.364^\circ$, hence again TIR does not happen.]

Regardless of whether the sound beam is incident from air into water or from water into air, by Snell’s law

$$\sin(\Theta_{\text{Water}}) = \left(\frac{v_{\text{Water}}}{v_{\text{Air}}}\right) \sin(\Theta_{\text{Air}})$$

where $\Theta_{\text{Air}}$ and $\Theta_{\text{Water}}$ are the angles of the sound beam in air and water, respectively, measured from the normal, and $v_{\text{Air}} = 346\text{m/s}$ and $v_{\text{Water}} = 1497\text{m/s}$ are the corresponding speeds. Since $v_{\text{Water}} > v_{\text{Air}}$ this implies that $\sin(\Theta_{\text{Water}}) > \sin(\Theta_{\text{Air}})$ and hence

$$\Theta_{\text{Water}} > \Theta_{\text{Air}}$$

That is, as a matter of general principle, the beam always has the greater angle to the normal when traveling in the medium with the greater speed. (And this is always true for any kind of wave being refracted between any two different media!)

So, if incident from air, the sound beam is refracted away from the normal upon entering the water; and if incident from water, the sound beam is refracted towards the normal upon entering the air (provided that refraction occurs at all, i.e., absent TIR). For these reasons answer (D) is correct and (E) is wrong.

**CP 1.07:** If a real object is placed more than two focal lengths away from a concave mirror ($f > 0$), then the image is

- (A) virtual, erect and enlarged in height relative to the object
- (B) virtual, erect and reduced in height relative to the object
- (C) real, inverted and reduced in height relative to the object
- (D) real, inverted and enlarged in height relative to the object
- (E) real, erect and reduced in height relative to the object
Answer: (C)

Being "more than two focal lengths away" means \( d > 2f \). From that, and from \( f > 0 \), it follows that

\[
\frac{1}{d} < \frac{1}{2(2f)} = \frac{1}{2} \left( \frac{1}{f} \right)
\]

(by taking the inverse of the inequality \( d > 2f \)). Therefore

\[
\frac{1}{d'} = \frac{1}{f} - \frac{1}{d} > \frac{1}{f} - \frac{1}{2} \left( \frac{1}{f} \right) = \frac{1}{2} \left( \frac{1}{f} \right)
\]

and hence [by taking the inverse of the inequality \( \frac{1}{d'} > \frac{1}{2} \left( \frac{1}{f} \right) \) and using \( d > 2f \) again] it follows that

\[
d' < 2f < d
\]

but also \( \frac{1}{d'} > 0 \) which implies \( d' > 0 \), i.e., the image is real.

From \( 0 < d' < d \) and \( m = -d'/d \) it then follows that

\[
|m| = \frac{d'}{d} < 1
\]

i.e., the image is reduced in absolute height, compared to the object; and it also follows that

\[
m = -\frac{d'}{d} < 0
\]

i.e., the image is inverted, relative to the object. So, the answer is (C).

Lastly, if you do not like general mathematical proofs, like the one above, you could also answer this question simply by plugging in some numbers for \( d \) and \( f \). Take, e.g., \( f = 4cm \) and \( d = 10cm \) so that \( d > 2f = 8cm \), i.e., the object is placed more than two focal lengths from mirror. Then, you’ll get \( d' = \frac{1}{f} - \frac{1}{d} \) and from that \( m = -\frac{d'}{d} = \frac{-20}{3} \) and hence \( |m| = \frac{20}{3} \) which implies \( d' > 0 \), i.e., the image is real.

Since \( m < 0 \), the image is inverted, relative to object

Since \( |m| < 1 \), the image is reduced in absolute height, relative to object.

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**CP 1.08:** Two non-parallel light rays initially converge to a single point on a flat screen so that the normal to the screen is enclosed between the two incident rays, as shown here:
Problem 1: Two non-parallel light rays initially converge to a single point on a screen. A rectangular block of glass is now placed somewhere in front of the screen, in the path of the light rays, so that the glass surface is parallel to the screen. Where is the new convergence point of the rays?

The rays will no longer converge on the screen. Instead, the convergence point will move away from the glass slab, as shown by the diagram. After leaving the glass, the rays are traveling in the same direction as they were before (because the glass is a slab with parallel faces). However, the rays have been displaced a bit. It's this displacement that causes the convergence point to move.

Problem 2: While standing on a lakeside pier, you see a fish in the water and decide to try some less common methods of snagging dinner. (a) If you try to snag the fish with a harpoon, where should you aim? (b) Where should you aim if you instead use a high-powered laser to zap the fish?

(a) To answer this question, you need to understand that the fish is not actually where you see it, due to refraction. Light rays coming from the fish bend away from the normal as they travel from water into air. As a result, you perceive the fish to be above where it actually is. Your harpoon won’t change direction when it enters the water, so in order to hit the actual fish, you should aim below where you see the image of the fish.

(b) Using the laser, you should aim directly at (the image of) the fish. Unlike an arrow, laser light will be refracted by the water. If you aim in the direction where you see the fish, the laser light will refract by exactly the same amount as the light coming from the fish to your eyes (just in reverse).

A slab of glass is now placed somewhere in front of the screen, in the path of the light rays, so that the slab’s two planar glass surfaces are parallel to the screen. The index of refraction (IoR) of the glass is greater than the IoR of the surrounding air. Where is the new convergence point of the rays?

(A) On the screen (unchanged);
(B) Toward the glass slab, in front of the screen (i.e., between slab and screen);
(C) Further away from the glass slab, behind (i.e., the right of) the screen;
(D) Inside the glass slab;
(E) Cannot be determined from the information given.

Answer: (C)

At the 1st refraction, when entering the slab, both rays are refracted towards the normal inside the slab, since $n_{\text{Glass}} > n_{\text{Air}}$. So, Ray 1 is bent upward and Ray 2 is bent downward, inside the slab, as shown in the figure above.

Then, at the 2nd refraction, when leaving the slab, both rays are refracted away from the normal (again because $n_{\text{Glass}} > n_{\text{Air}}$) so that the 2nd refraction is exactly ”un-doing” the change of direction the two rays suffered as a result of the 1st refraction. This happens as a result of Snell’s law and as a result of the fact that, inside the slab, the angle of refraction at the left surface equals angle of incidence at the right surface (... and these two angles are equal because the left and right slab surfaces are parallel planes).

As a consequence, (a) the direction of Ray 1 exiting the slab (to the right of slab) is parallel to the direction of Ray 1 entering the slab, but shifted upward relative to the entering Ray 1; and (b) likewise the direction of Ray 2 exiting the slab (to the right of slab) is parallel to the direction of Ray 2 entering the slab, but shifted downward relative to the entering Ray 2.

In the absence of the slab, without refraction, both rays would converge to their point of intersection located exactly on the screen, as indicated by dotted lines in the figure. Hence,
as a result of the upward shift of Ray 1 and of the downward shift of Ray 2 (both caused by the refractions at the slab), this point of convergence is shifted to the right, i.e., behind the screen, as shown in figure.

**CP 1.09:** If a virtual object \( (d < 0) \) is positioned at an absolute distance \(|d|\) less than the absolute focal length \(|f|\) from a divergent lens \((f < 0)\), then the image is

(A) virtual, erect and enlarged in height relative to the object  
(B) virtual, erect and reduced in height relative to the object  
(C) real, inverted and reduced in height relative to the object  
(D) real, inverted and enlarged in height relative to the object  
(E) real, erect and enlarged in height relative to the object

**Answer:** (E)

If \(0 > d = -|d|\) (virtual object!) and \(0 > f = -|f|\) (divergent lens!) and \(|d| < |f|\), then:

\[
\frac{1}{|d|} > \frac{1}{|f|} \quad \text{and thus} \quad 1/f - 1/d = -1/|f| - (-1/|d|) = 1/|d| - 1/|f| > 0 .
\]

Hence

\[
d' = (1/f - 1/d)^{-1} = (1/|d| - 1/|f|)^{-1} = \left[ \frac{|f| - |d|}{|d||f|} \right]^{-1} = |d| \times \left[ \frac{|f|}{(|f| - |d|)} \right] > 0;
\]

and \(d' > 0\) means we have a **real** image. Also, since \(|f| > |f| - |d| > 0\), we have

\[
\left[ \frac{|f|}{(|f| - |d|)} \right] > 1 , \quad \text{or} \quad d' = |d| \times \left[ \frac{|f|}{(|f| - |d|)} \right] > |d| ;
\]

and therefore

\[
m = -d'/d = d'/|d| = \left[ \frac{|f|}{(|f| - |d|)} \right] > 1 .
\]

Now, \(m > 1\) implies \(|m| > 1\), *i.e.* the image is **enlarged**, and it also implies \(m > 0\), *i.e.* the image is **erect**.

Lastly, if you do not like general mathematical proofs, like the one above, you could also answer this question simply by plugging in some numbers for \(d\) and \(f\). Take, *e.g.* \(f = -4\text{cm}\) and \(d = -3\text{cm}\), chosen so that \(d < 0\) and \(f < 0\) and \(|d| < |f|\). Then, you’ll get \(d' = (1/f - 1/d)^{-1} = [-(1/4) + (1/3)]^{-1}\text{cm} = +12\text{cm}\) and from that \(m = -d'/d = -(+12)/(-3) = +4\). Therefore:

Since \(d' > 0\), the image is real.

Since \(m > 0\), the image is erect, relative to object.

Since \(|m| > 1\), the image is enlarged in absolute height, relative to object.

**CP 1.10:** If a real object is placed in front of a convex mirror \((f < 0)\), then the image is
(A) virtual, erect and enlarged in height relative to the object
(B) virtual, erect and reduced in height relative to the object
(C) real, erect and reduced in height relative to the object
(D) real, inverted and enlarged in height relative to the object
(E) real, erect and enlarged in height relative to the object

Answer: (B)

If $0 < d = +|d|$ (real object!) and $0 > f = -|f|$ (convex mirror!), then:

$$\frac{1}{d'} = \frac{1}{f} - \frac{1}{|d|} = -(1/|f| + 1/|d|) = -\frac{|d| + |f|}{|f||d|} < 0;$$

thus

$$d' = -\frac{|f||d|}{|d| + |f|} = -|d| \times \left[\frac{|f|}{|d| + |f|}\right] < 0;$$

and $d' < 0$ means we have a virtual image. Also, since $|f| < |d| + |f|$, we have

$$\left[\frac{|f|}{(|f| + |d|)}\right] < 1,$$

or $|d'| = |d| \times \left[\frac{|f|}{(|f| + |d|)}\right] < |d|$;

and therefore

$$m = -d'/d = |d'|/|d| = \left[\frac{|f|}{(|f| + |d|)}\right] < 1; \quad \text{but also} \quad m > 0.$$

Now, $0 < m < 1$ implies $|m| < 1$, i.e. the image is reduced; and $m > 0$ means that the image is erect.

Lastly, if you do not like general mathematical proofs, like the one above, you could also answer this question simply by plugging in some numbers for $d$ and $f$. Take, e.g. $f = -4\text{cm}$ and $d = +3\text{cm}$, chosen so that $d > 0$ and $f < 0$. Then, you’ll get $d' = (1/f - 1/d)^{-1} = \left[-(1/4) - (1/3)\right]^{-1}\text{cm} = -(12/7)\text{cm}$ and from that $m = -d'/d = -(-12/7)/(+3) = +(4/7)$. Therefore:

Since $d' < 0$, the image is virtual.

Since $m > 0$, the image is erect, relative to object.

Since $|m| < 1$, the image is reduced in absolute height, relative to object.

**CP 1.11:** Two identical solid blocks $S$ made from the same transparent material are immersed in two different liquids $A$ and $B$. A ray of light strikes each block at the same angle of incidence, as shown. According to the figure below, what is the relative magnitude of the indices of refraction of the solid blocks, $n_S$, and the two liquids, $n_A$ and $n_B$?
First off, before you read any further, make a large (re-)drawing of the figure above in which the normal to the top surface of each solid block, and the corresponding angle of incidence $\Theta$ and angles of refraction, $\Theta'_A$ or $\Theta'_B$, respectively, at the top surface, are clearly shown.

If you really know your stuff, this quick ’n dirty argument will give you the answer, fast:

1. As shown in the figure, the initial incident ray (above block $S$) is refracted by the block towards the normal in both liquids $A$ and $B$; hence $n_S > n_A$ and $n_S > n_B$.

2. As also shown in the figure, in liquid $A$, the incident ray is refracted (i.e., ”bent” by the block) towards the normal more strongly than in liquid $B$. Hence, angle of incidence and index of refraction (IoR) $n_S$ of the block $S$ being the same in both cases, we must have $n_A < n_B$.

3. So, combining the three inequalities from (1) and (2), we must have $n_A < n_B < n_S$ which is Answer (D).

If you don’t know your stuff quite yet, you are strongly advised to work through and absorb the following more detailed and mathematically rigorous argument:

At the top surfaces of both blocks, compare the relative sizes of: (1) angle of incidence $\Theta$ (same in either liquid), (2) angle of refraction $\Theta'_A$ inside block in liquid $A$, (3) angle of refraction $\Theta'_B$ inside block in liquid $B$. Namely, with normals to top surfaces carefully drawn in and angles carefully labeled, you can read off from the figure that

$$\Theta'_A < \Theta'_B < \Theta.$$
Hence, since the sine increases with angle (for angles between $0^\circ$ and $90^\circ$):

$$\sin(\Theta_A') < \sin(\Theta_B') < \sin(\Theta).$$

Thus, after dividing both inequalities by $\sin(\Theta)$ on both sides:

$$\frac{\sin(\Theta_A')}{\sin(\Theta)} < \frac{\sin(\Theta_B')}{\sin(\Theta)} < 1$$

But, by Snell’s law, $n_S\sin(\Theta_A') = n_A\sin(\Theta)$ and $n_S\sin(\Theta_B') = n_B\sin(\Theta)$, or equivalently:

$$\frac{\sin(\Theta_A')}{\sin(\Theta)} = \frac{n_A}{n_S} \quad \text{and} \quad \frac{\sin(\Theta_B')}{\sin(\Theta)} = \frac{n_B}{n_S}$$

Thus replacing sine-ratios in the foregoing inequalities by corresponding IoR-ratios:

$$\frac{n_A}{n_S} < \frac{n_B}{n_S} < 1$$

Multiplying both these inequalities by $n_S$ then gives Answer (D).

**CP 1.12:** Two identically shaped solid blocks, $S$ and $T$, made from the two different transparent materials, are immersed in the same liquid $L$. A ray of light strikes each block at the same angle of incidence, as shown. According to the figure below, what is the relative magnitude of the indices of refraction of the solid blocks, $n_S$ and $n_T$, and liquid, $n_L$?

(A) $n_L < n_S < n_T$;
(B) $n_S < n_L < n_T$;
(C) $n_S < n_T < n_L$;
(D) $n_L < n_T < n_S$;
(E) $n_T < n_L < n_S$. 
Answer: (D)

First off, before you read any further, *make a large (re-)drawing* of the figure above wherein the normal to the top surface of each solid block, and the corresponding angle of incidence $\Theta$ and angles of refraction, $\Theta'_S$ or $\Theta'_T$, respectively, at the top surface, are clearly shown.

If you really *know your stuff*, this quick 'n dirty argument will give you the answer, fast:

1. As shown in the figure, the initial incident ray (above block $S$ or $T$) is refracted by the block *towards the normal* by both blocks $S$ and $T$; hence $n_S > n_L$ and $n_T > n_L$.

2. As also shown in the figure, in block $S$, the incident ray is refracted (*i.e.*, “bent” by the block) towards the normal more strongly than in block $T$. Hence, angle of incidence and index of refraction (IoR) $n_L$ of the liquid $L$ being the same in both cases, we must have $n_S > n_T$.

3. So, combining the three inequalities from (1) and (2), we must have $n_L < n_T < n_S$ which is Answer (D).

If you *don’t know your stuff quite yet*, you are strongly advised to work through and absorb the following more detailed and mathematically rigorous argument:

At the top surfaces of both blocks, compare the relative sizes of: (1) angle of incidence $\Theta$ (same in either liquid), (2) angle of refraction $\Theta'_S$ inside block $S$, (3) angle of refraction $\Theta'_T$ inside block $T$. Namely, with normals to top surfaces carefully drawn in and angles carefully labeled, you can read off from the figure that

$$\Theta > \Theta'_T > \Theta'_S$$

Hence, since the sine increases with angle (for angles between $0^\circ$ and $90^\circ$):

$$\sin(\Theta) > \sin(\Theta'_T) > \sin(\Theta'_S).$$

Thus, after dividing both inequalities by $\sin(\Theta)$ on both sides:

$$1 > \frac{\sin(\Theta'_T)}{\sin(\Theta)} > \frac{\sin(\Theta'_S)}{\sin(\Theta)}.$$

But, by Snell’s law, $n_T\sin(\Theta'_T) = n_L\sin(\Theta)$ and $n_S\sin(\Theta'_S) = n_L\sin(\Theta)$, or equivalently:

$$\frac{\sin(\Theta'_T)}{\sin(\Theta)} = \frac{n_L}{n_T} \quad \text{and} \quad \frac{\sin(\Theta'_S)}{\sin(\Theta)} = \frac{n_L}{n_S}.$$

Thus replacing sine-ratios in the foregoing inequalities by corresponding IoR-ratios:

$$1 > \frac{n_L}{n_T} > \frac{n_L}{n_S}.$$

Dividing both these inequalities by $n_L$ then gives

$$\frac{1}{n_L} > \frac{1}{n_T} > \frac{1}{n_S}.$$

Taking the inverse on both sides of both of these two inequalities thus gives Answer (D). (Recall here that, if $1/a > 1/b$ for positive $a$ and $b$, then $a < b$)