A Compound Microscope

This is an example for image formation in a compound microscope, similar to the one done in the lecture. The following gives a step-by-step description how to analyze the image formation process. Each of the 14 solution steps is illustrated by a drawing (figure), shown in a separate pdf- and ppt-file. You should download this figure file before starting on the problem and solution below. Look through the sequence of figures in this file to get an overview of the solution, when working through the 14 solution steps below.

You must always begin the solution to problems like this by making a large, clean drawing. Start with the information given in the problem statement. Then fill in more details, as you work your way, step by step, through the solution. The (pdf/ppt) figure file accompanying this solution outline illustrates how such a drawing should be made, as you proceed with each step of your calculation.

Problem Statement:
A compound microscope is built with an objective lens (=Lens 1) of focal length f₁=0.65cm, and an eyepiece lens (=Lens 2) of focal length f₂=2.50cm with Lens 2 to the right of Lens 1.

A small object (Obj1) of size 0.032cm is placed 0.70cm to the left of Lens 1.

The distance between the two lenses is adjusted so that the final image (Img2), as seen by the eye, looking through the eyepiece, is located at the eye's near point distance, 20cm to the left of Lens 2.

State or calculate all image parameters: \(d_1, \ d_1', \ d_2, \ d_2', \ h_1, \ h_1', \ h_2, \ h_2'.\) Calculate the lens-to-lens distance \(L.\)

Calculate the angle \(\Theta_e\) subtended at the Eye by the final image, Img2, when viewed through the eyepiece Lens 2.

Calculate the angle \(\Theta_{ref}\) subtended at the Eye by the original object, placed at the near point, 20cm in front of the eye and viewed without microscope.

From \(\Theta_e\) and \(\Theta_{ref}\) calculate the angular magnification \(M_e = \Theta_e / \Theta_{ref}.\)

Solution Steps:
1. Draw lenses, original source object (Obj1) and receiver of final image (Eye) on optical axis, with Obj1 drawn as an arrow pointing upward.

2. Trace the general ray direction through lenses: from the source (Obj1) to the receiver (Eye).
3. Mark “in”, “out”, “not in” and “not out” for each lens, using ray traced through both lenses to figure out which side of each lens is “in”, which side is “out”.

4. Mark focal point F₁ for Lens 1 on optical axis, relative to Obj1 and Lens 1. Since f₁>0, F₁ must be on the “in” side of Lens 1. So, F₁ is to the left of Lens 1. Also: |d₁|>|f₁|, so Obj1 is further away than F₁ from Lens 1.

5. Draw and mark object distance d₁ as an arrow from Lens 1 to Obj1. Since Obj1 is, by definition, on the “in” side of Lens 1, Obj1 is a real object and d₁>0: d₁ = +0.70cm

6. Calculate image distance d₁’ of Img1 from Lens 1:
   \[ d₁' = \left( \frac{1}{f₁} - \frac{1}{d₁} \right)^{-1} = \left( \frac{1}{0.65} - \frac{1}{0.70} \right)^{-1}cm = \frac{9.10cm}{cm} > 0 \]
   
   Calculate the lateral magnification of Lens 1:
   \[ m₁ = -\frac{d₁'}{d₁} = -\frac{9.10}{0.70} = -13.0 < 0, \text{ since both } d₁ > 0, d₁'>0. \]
   
   Since d₁'>0, Img1 is real, i.e., on the “out” side of Lens 1. Since m₁'<0, Img1 is inverted relative to Obj1. Draw Img1 on the optical axis accordingly: as a downward arrow to the right of Lens 1.

   Draw and mark image distance d₁' as an arrow from Lens 1 to Img1.

   Note: at this point, you may not know yet whether Img1 is placed to the left or to the right of Lens 2!? Pick one of the two choices! If it later turns out to be wrong you can correct the drawing then. The pdf-Figure for Step 6 shows Img1 to the left, i.e., on the “in” side of Lens 2, which turns out to be the correct choice of placement (see Step 8).

7. Draw Img2, the image of Lens 2. Since Img 2 is the final image, to be seen by the Eye, it must be placed in front of the Eye, i.e., at least a near-point distance away, to the left of the Eye and Lens 2. Since this places Img2 on the “not out” side of Lens 2, Img2 is a virtual image and its image distance d₂’ must be negative:
   \[ d₂' = -|d₂'| = -d_{\text{near}} = -20.0cm < 0 \]
   
   Draw and mark image distance d₂’ as an arrow from Lens 2 to Img2.

   Note: at this point, you may not know yet whether Img2 has the same or opposite (inverted or erect?) orientation as Obj2=Img1!? Pick one of the two choices! If it later turns out to be wrong you can correct the drawing then. The pdf-Figure for Step 7 shows Img2 pointing down, i.e., having the same orientation as Obj2=Img1, which turns out to be the correct choice of orientation (see Step 8).

8. Calculate object distance d₂ of Obj2=Img1 from Lens 1:
\[ d_2 = \left( \frac{1}{f_2} - \frac{1}{d_2} \right)^{-1} = \left( \frac{1}{2.50} - \frac{1}{(-20.0)} \right)^{-1} \text{cm} = +2.22 \text{cm} > 0 \]

Calculate the lateral magnification of Lens 2:
\[ m_2 = -\frac{d_2'}{d_2} = -\left(\frac{-20}{2.22}\right) = +9.00 > 0, \text{ since } d_2 > 0, \ d_2' < 0. \]

Since \( d_2 > 0 \), Obj2 is real, i.e., located on the “in” side of Lens 2. So Obj2 is to the left of Lens 2. If you did place Img1=Obj2 incorrectly relative to Lens 2 (to the right of Lens 2), in Step 6, correct it now: Img1=Obj2 and Lens 2 must be drawn such that Obj2 is to the left of Lens 2.

Draw and mark object distance \( d_2 \) as an arrow from Lens 2 to Img2.

Since \( m_2' > 0 \), Img2 is not inverted relative to Obj2: Img 2 must have the same orientation as Obj2=Img1. So, if you did draw Img2 incorrectly (as an upward-pointing arrow), in Step 7, correct it now: Img2 must be drawn as an arrow pointing downward.

9. Calculate the Lens1-Lens2 distance \( L \):
\[ L = d_1' + d_2 = +9.10 \text{cm} + 2.22 \text{cm} = 11.32 \text{cm} \]
Draw \( L \) between Lens1 and Lens2.

10. State the original object height \( h_1 \) for Obj1 and calculate image height \( h_1' \) for Img1:
\[ h_1 = 0.032 \text{cm} \]
\[ h_1' = m_1 h_1 = \left(\frac{-d_1'}{d_1}\right) h_1 = -\left(\frac{9.10}{0.70}\right)(0.032 \text{cm}) = -0.416 \text{cm} \]
Mark \( h_1 \) and \( h_1' \) in the drawing.

11. State the object height \( h_2 \) for Obj2. Since Img1 from Lens1 serves as the object for Lens2, Obj2, see drawing:
\[ h_2 = h_1' = -0.416 \text{cm} \]
Then calculate image height \( h_2' \) for Img2:
\[ h_2' = m_2 h_2 = \left(\frac{-d_2'}{d_2}\right) h_2 = -\left(\frac{-20}{2.22}\right)(-0.416 \text{cm}) = -3.74 \text{cm} \]
Mark \( h_2 \) and \( h_2' \) in the drawing.

12. Calculate the angle \( \Theta_e \) subtended at the Eye by the final image, Img2, when viewed through the eyepiece Lens 2. The height, \( h_e \), and approximate distance from Eye(piece), \( d_e \), for that final image are given
\[ h_e = \left| h_2' \right| = 3.74 \text{cm} \quad \text{and} \quad d_e = \left| d_2' \right| = 20.0 \text{cm} \]
Thus:
\[ \Theta_e = \frac{\tan(\Theta_e)}{h_e} = \frac{d_e}{d_2} = \frac{\left| h_2' \right|}{\left| d_2' \right|} = \frac{3.74}{20} = 0.187 \text{ rad} \]
Mark \( h_e \), \( d_e \), \( \Theta_e \) in the microscope drawing. Draw \( h_e \), \( d_e \), \( \Theta_e \) in a separate drawing, if needed for your own clarification.

13. Calculate the angle \( \Theta_{ref} \) subtended at the Eye by the original object, placed at the eye’s near point, in front of the eye and viewed without microscope. The
height, $h_0$, and distance from Eye(piece), $d_{\text{near}}$, for the original object are given

$$h_0 = h_1 = 0.032\text{cm} \quad \text{and} \quad d_{\text{near}} = 20.0\text{cm}$$

Thus:

$$\Theta_{\text{ref}} \approx \tan(\Theta_{\text{ref}}) = \frac{h_0}{d_{\text{near}}} = \frac{h_1}{d_{\text{near}}} = \frac{0.032}{20} = 0.0016 \text{ rad}$$

Draw $h_0$, $d_{\text{near}}$, $\Theta_{\text{ref}}$ in a separate drawing.

14. Calculate the angular magnification:

$$M_{\Theta} = \frac{\Theta_e}{\Theta_{\text{ref}}} = \frac{0.187}{0.0016} = 117.$$  

Exercises:

1. Repeat the foregoing calculation using the same input parameters, except that the final image, Img2, is now placed very far away from the Eye(piece), at a distance

$$d_e = |d_2'| = \infty \quad \text{so that} \quad \frac{1}{d_2'} = 0.$$  

2. Repeat the foregoing calculations, both for $d_e = |d_2'| = 20\text{cm}$ and $d_e = |d_2'| = \infty$, using the same input parameters, except that Lens 2 is replaced by a divergent lens with focal length $f_2 = -2.50\text{cm}$.

What happens to $d_2$ and $m_2$ now?

Do you need to modify the placement of Obj2 relative to Lens2 in your drawing? Is Obj2 = Img1 to the right or to the left of Lens2 now?

Note: Obj2 may be a virtual object now, even though Img1 is still a real image.

Do you need to modify the orientation of the final image, Img2, in your drawing? Is Img2 an upward or a downward arrow now?

How does $M_{\Theta}$ change when compared to the case $f_2 = +2.50\text{cm}$?

3. Use the approximate angular magnification formula given in the textbook which, in our notation and ignoring the sign, reads as follows:

$$M_{\Theta} = \frac{d_{\text{near}} d_1'}{|f_1 f_2|}$$

[The translation Our Notation == Textbook Notation is as follows:

$$d_{\text{near}} = N \quad d_1' = d_i \quad f_1 = f_{\text{objective}} \quad f_2 = f_{\text{eyepiece}}.$$]

How do the results from the textbook formula compare to the exact results, calculated from $M_{\Theta} = \frac{\Theta_e}{\Theta_{\text{ref}}}$?
Compound Microscope: Step 1

Object 1

Lens 1

Lens 2

Eye
Compound Microscope: Step 2

General Direction of Light Rays Passing thru Lenses

Object 1

Lens 1

Lens 2

Eye
Compound Microscope: Step 3

1: in 
1: not out
1: not in 
1: out

2: in 
2: not out
2: not in 
2: out

General Direction of Light Rays Passing thru Lenses
Compound Microscope: Step 4
Compound Microscope: Step 5

1: in
1: not out

1: not in
1: out

2: in
2: not out

2: not in
2: out
Compound Microscope: Step 6

1: in
1: not out
1: not in
1: out

2: in
2: not out
2: not in
2: out

$d_1$ $d_1'$

$F_1$
Compound Microscope: Step7

1: in
1: not out
1: not in
1: out

2: in
2: not out
2: not in
2: out

$d_1$
$d_1'$
$F_1$

d_2'$

Eye
Compound Microscope: Step 8
Compound Microscope: Step9

1: in
1: not in
1: out

2: in
2: not in
2: out

\( \text{d}_1 \)
\( \text{d}_1' \)

\( \text{d}_2 \)
\( \text{d}_2' \)

\( \text{F}_1 \)
\( \text{F}_2 \)

L
Compound Microscope: Step10

- **1**: in
- **1**: not in
- **1**: not out
- **1**: out

- **2**: in
- **2**: not in
- **2**: not out
- **2**: out

Diagram showing distances and focal points:
- \( h_1 \) and \( h_1' \)
- \( d_1 \) and \( d_1' \)
- \( F_1 \) and \( F_2 \)
- \( d_2 \) and \( d_2' \)

Eye view and image representation.
$h_0 = h_1$

$d_{\text{near}}$

$\theta_{\text{ref}}$

Eye

Compound Microscope: Step 13
\[
\begin{align*}
M_\Theta &= \frac{\theta_e}{\theta_{\text{ref}}} \\
|h_{2'}| &= h_e \\
|d_{2'}| &= d_e \\
h_o &= h_1 \\
d_{\text{near}} &= \theta_{\text{ref}}
\end{align*}
\]
\[ h_1' = h_2 \]

\[ d_1 = d_1' \]

\[ F_1 \]

\[ F_2 \]

\[ d_2' \]

\[ \theta_e \]

Compound Microscope