Chapter 9: Linear Momentum and Collisions

Answers to Even-Numbered Conceptual Questions

16. (a) Assuming a very thin base, we conclude that the center of mass of the glass is at its geometric center of the glass. (b) In the early stages of filling, the center of mass is below the center of the glass. When the glass is practically full, the center of mass is again at the geometric center of the glass. Thus, as water is added, the center of mass first moves downward, then turns around and moves back upward to its initial position.

Answers to Even-Numbered Conceptual Exercises

10. The center of mass is higher than the midway point between the tip of the stalactite and the cave floor. The reason is that as the drops fall, their separations increase (see Conceptual Checkpoint 2-5). With the drops more closely spaced on the upper half of their fall, the center of mass is shifted above the halfway mark.

14. The two halves of the triangle have the same mass, but the left side is, on average, farther from the fulcrum than the right side. As a result, the center of mass is to the left of the fulcrum, and hence the triangle will tip to the left when released.

Solutions to Problems

45. Picture the Problem: One end of the rope is lifted at constant speed.

Strategy: Use equation 9-15 to find the position of the center of mass and equation 9-16 to find the velocity of the center of mass of the rope. When writing subscripts for the variables, let \( f \) = on the floor, \( nf \) = not on the floor, and \( L \) = length of the rope. The graphs should all end at \( t = L/v = (2.00 \text{ m})/0.810 \text{ m/s} = 2.47 \text{ s} \), at which time the rope will be entirely off the floor and the velocity of its center of mass will be 0.810 m/s upward. The top of the rope is at position \( vt \) during the lift, so the center of mass of the above floor portion of the rope is halfway between zero and \( vt \), and the fraction of the rope that is above the floor is \( m_{nf} = (vt/L) M \), where \( M \) is the total mass of the rope.

Solution: 1. Use equation 9-15 to find the position of the center of mass of the rope:

2. Use equation 9-16 to find the velocity of the center of mass of the rope:

3. The plots of \( Y_{cm} \) and \( V_{cm} \) as a function of time are shown above.

Insight: The position of the center of mass varies as the square of the time at first but would become linear with a slope of 0.810 m/s as soon as the last bit of rope left the floor (at \( t = 2.47 \text{ s} \)).
47. **Picture the Problem:** The physical arrangement for this problem is depicted in the figure at right.

**Strategy:** Before the string breaks, the reading on the scale is the total weight of the ball and the liquid. After the string breaks, the ball falls with constant speed, so that the center of mass of the ball–liquid system does not accelerate.

**Solution:**  
1. (a) Find the total weight of the ball and the liquid: 
   \[ Mg = (1.20 + 0.150 \text{ kg}) \times 9.81 \text{ m/s}^2 = 13.2 \text{ N}. \]

2. (b) After the string breaks, the reading is 13.2 N. Because the center of mass of the ball-liquid system undergoes no net acceleration, the scale must not exert any net force on it (Newton’s Second Law), and by Newton’s Third Law it exerts no net force on the scale. Therefore, the reading will not change.

**Insight:** If the ball were to accelerate, the center of mass of the ball-liquid system would accelerate downward, and the scale force would decrease. Another way of looking at the problem is to realize that the water must exert exactly the same upward force on the ball (1.47 N) when it is at rest as when it is falling at constant speed.

48. **Picture the Problem:** The two blocks are connected by a string and hang vertically from a spring.

**Strategy:** Use Newton’s Second Law to determine the force exerted by the spring before the string is cut. Then sum the forces on the two blocks immediately after the string is cut to find the net force. Finally, use Newton’s Second Law again to find the acceleration of the two-block system just after the string is cut.

**Solution:**  
1. (a) Before the string is cut, the force of gravity is countered by the force of the spring, so the spring force at this stretch distance is \(2mg\). Just after the string is cut, the upper block experiences a force of 
   \[ F_s + F_g = 2mg - mg = mg, \]
   and the lower block experiences a force of 
   \[ F_g = -mg. \]
   The net force acting on the two-block system is 
   \[ F_{\text{net, ext}} = mg - (mg) = 0. \]

2. (b) Since \( F_{\text{net, ext}} = MA_{\text{cm}} = 0 \), we conclude that \( A_{\text{cm}} = 0 \).

**Insight:** While the top block moves upward it experiences a smaller and smaller force from the spring. When it reaches the unstretched position of the spring, the force on it is \(-mg\), so that the net force on the two block system is \(-2mg\) and the acceleration of the two blocks is \(-g\).

59. **Picture the Problem:** The falling raindrops are stopped by the upward force from the ground.

**Strategy:** Calculate the rate \(\Delta m/\Delta t\) at which water is delivered to a square meter of the ground, and then use the thrust equation (9-19) to estimate the force.

**Solution:**  
1. Find the rate at which the water fell over 1.0 m²:

2. Find the thrust from equation 9-19: 
   \[ \text{thrust} = \left( \frac{\Delta m}{\Delta t} \right)v = (0.024 \text{ kg/s})(10 \text{ m/s}) = 0.24 \text{ N}. \]

**Insight:** Since a Newton is about \(\frac{1}{4}\) lb, this force is about \(\frac{1}{16}\) lb or about one ounce on a square meter. However, if you allow all 31 inches to pool on top of the square meter it will weigh 7.7 kN = 1700 lb = 0.87 ton!
71. **Picture the Problem:** The coasting rocket explodes into two pieces of equal mass that are ejected at 45.0° to the vertical.

**Strategy:** Assume gravity is the only force acting on the rocket after it is launched. Find its speed after rising for 2.50 s, then use conservation of momentum in the vertical direction and the principles of center-of-mass motion to answer the questions.

**Solution:** 1. (a) Use equation 4-6 to find the speed of the rocket before the explosion:

\[
v_{iy} = v_{iy} - gt = (44.2 \text{ m/s}) - (9.81 \text{ m/s}^2)(2.50 \text{ s})
\]

\[
v_{iy} = 19.68 \text{ m/s}
\]

2. Set \( p_{ix} = p_{fx} \) and solve for \( v_{ix} \) of each piece:

\[
2mv_{ix} = 2mv_i \sin 45.0^\circ
\]

\[
v_{ix} = \frac{v_{iy} \sin 45.0^\circ}{\sin 45.0^\circ} = \frac{19.68 \text{ m/s} \sin 45.0^\circ}{\sin 45.0^\circ} = 27.8 \text{ m/s}
\]

3. (c) Before the explosion \( V_{cm} = [19.7 \text{ m/s}] \hat{y} \) Since the momentum of the system is the same after the explosion, and the total mass has not changed, \( V_{cm} = [19.7 \text{ m/s}] \hat{y} \) after the explosion too.

4. (d) The only force acting on the system before and after the explosion is gravity. Therefore, \[ A_{cm} = [-9.81 \text{ m/s}^2] \hat{y} \]

**Insight:** Momentum is conserved in the \( x \) direction, as well, as guaranteed by the fact that each piece has the same speed and the same angle from vertical. The momentum in the \( x \) direction is zero both before and after the explosion.

73. **Picture the Problem:** The books are arranged in a stack as depicted at right, with book 1 on the bottom and book 4 at the top of the stack.

**Strategy:** It is helpful to approach this problem from the top down. The center of mass of each set of books must be above or to the left of the point of support. Find the positions of the centers of mass for successive stacks of books to determine \( d \). Measure the positions of the books from the right edge of book 1 (right hand dashed line in the figure).

**Solution:** 1. The center of mass of book 4 needs to be above the right end of book 3.

\[
d_1 = \frac{L}{2}
\]

2. The result of step 1 means that the center of mass of book 3 is located at \( L/2 + L/2 = L \) from the right edge of book 1.

3. The center of mass of books 4 and 3 needs to be above the right end of book 2:

\[
d_2 = X_{cm43} = \frac{m[L/2] + m[L]}{2m} = \frac{3L}{4}
\]

4. The result of step 3 means that the center of mass of book 2 is located at \( 3L/4 + L/2 = 5L/4 \).

5. The center of mass of books 4, 3, and 2 needs to be above the right end of book 1:

\[
d_3 = X_{cm432} = \frac{m[L/2] + m[L] + m[5L/4]}{3m} = \frac{11L}{12}
\]

6. The result of step 3 means that the center of mass of book 1 is located at \( 11L/12 + L/2 = 17L/12 \).

7. The center of mass of all four books needs to be above the right edge of the table:

\[
d = X_{cm4321} = \frac{m[L/2] + m[L] + m[5L/4] + m[17L/12]}{4m} = \frac{25L}{24}
\]

**Insight:** We will explore more about static equilibrium problems such as these in Chapter 11. If you examine the overhang of each book you find an interesting series: \[ d = \frac{L}{2} + \frac{L}{4} + \frac{L}{6} + \frac{L}{8} = \frac{25L}{24} \]. The series gives you a hint about how to predict the overhang of even larger stacks of books!
Chapter 10: Rotational Kinematics and Energy

Answers to Even-Numbered Conceptual Questions

2. Yes. In fact, this is the situation whenever you drive in a circular path with constant speed.

Answers to Even-Numbered Conceptual Exercises

10. The center of the outer quarter moves in a circle that has twice the radius of a quarter. As a result, the linear distance covered by the center of the outer quarter is twice the circumference of a quarter. Therefore, if the outer quarter rolls without slipping, it must complete two revolutions.

14. The ranking is as follows: case 1 < case 2 < case 3. To be specific, the moment of inertia for case 1 (x axis) is \( I = (9.0 \text{ kg})(1.0 \text{ m})^2 = 9.0 \text{ kg} \cdot \text{m}^2 \); for case 2 (y axis) it is \( I = (2.5 \text{ kg})(2.0 \text{ m})^2 = 10.0 \text{ kg} \cdot \text{m}^2 \); and for case 3 (z axis) it is \( I = (9.0 \text{ kg})(1.0 \text{ m})^2 + (2.5 \text{ kg})(2.0 \text{ m})^2 = 19.0 \text{ kg} \cdot \text{m}^2 \).

16. The ranking is as follows: disk 1 = disk 2 = disk 3. All uniform disks finish the race in the same time, regardless of their mass and/or radius.

Solutions to Problems

23. Picture the Problem: The saw blade rotates about its axis, slowing its angular speed at a constant rate until it comes to rest.

Strategy: Use the kinematic equations for rotation (equations 10-8 through 10-11) to find the angular acceleration of the saw blade and the angle through which the blade spins during this interval. Then use equation 10-2 to convert the angular distance to a linear distance.

Solution: 1. (a) Solve equation 10-8 for \( \alpha \) :
\[
\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - \left( \frac{4830 \text{ rev/min} \times 1 \text{ min/60 s}}{2.00 \text{ s}} \right)}{2.00 \text{ s}} = -40.3 \text{ rev/s}^2
\]
2. (b) Use equation 10-9 to find \( \Delta \theta \) :
\[
\Delta \theta = \frac{1}{2} \left( \omega + \omega_0 \right) t = \frac{1}{2} \left[ 0 + \frac{4830 \text{ rev/min} \times 1 \text{ min/60 s}}{2.00 \text{ s} \times 1 \text{ min/60 s}} \right] = 80.5 \text{ rev}
\]
3. Convert \( \Delta \theta \) to \( s \) :
\[
s = r\Delta \theta = \left( \frac{1}{2} 10.0 \text{ in/12 in/ft} \right) (80.5 \text{ rev} \times 2\pi \text{ rad/rev}) = 211 \text{ ft}
\]
4. (c) The blade completes exactly 80.5 revolutions, so a point on the rim ends up exactly opposite of where it started. Its displacement is therefore one blade diameter or 10.0 in.

Insight: If the blade had completed an integer number of revolutions, a point on the rim would end up exactly where it began and the displacement would be zero even though the distance it travels is hundreds of feet.
30. **Picture the Problem**: Jeff clings to a vine and swings along a vertical arc as depicted in the figure at right.

**Strategy**: Use equation 10-13 to find Jeff’s centripetal acceleration and equation 10-14 to find his tangential acceleration. Add these two perpendicular vectors to find the total acceleration.

**Solution**: 1. Apply equation 10-13 directly:

2. Apply equation 10-14 directly:

3. Add the two perpendicular vectors:

4. Find the angle $\phi$:

**Insight**: The angle $\phi$ will increase with Jeff’s speed if his angular acceleration remains constant because $a_c$ depends on the square of the tangential speed.

36. **Picture the Problem**: The ball moves in a circle of constant radius at constant speed.

**Strategy**: The motion is approximately horizontal so we can neglect the fact that the rope would be inclined a little bit below horizontal in order to support the weight of the ball. Set the rope tension equal to the centripetal force required to keep the ball moving in a circle and solve for the angular speed.

**Solution**: 1. (a) Set the string force $F = ma_c$ and solve for $\omega$:

2. (b) Since $\omega$ is inversely proportional to $r$, the maximum angular velocity will increases if the rope is shortened.

**Insight**: This is a fairly weak rope, since 11 N is only 2.5 lb. Still, the problem illustrates well how the centripetal force increases linearly with the distance from the rotation axis. Decreasing $r$ decreases the force, or allows a higher $\omega$ for the same amount of force.

52. **Picture the Problem**: The basketball rolls without slipping at constant speed on the level floor.

**Strategy**: Use equation 10-17 and the moment of inertia of a hollow sphere to find the rotational kinetic energy of the ball. Then use equation 7-6 to find the translational kinetic energy, and sum the two to find the total kinetic energy. Use a ratio to find out what fraction of the ball’s energy is rotational kinetic energy.

**Solution**: 1. (a) Apply equations 10-17 and 7-6 to find $K_r$, $K_t$, and the total $K$:

2. Calculate the ratio $K_r/K$:

3. (b) The answer to part (a) will stay the same if the linear speed of the ball is doubled to $2v$, because the ratio is independent of speed, radius, and mass.

**Insight**: The ratio is a constant because when an object rolls without slipping there is a direct relationship between its translation and rotation: $v_t = r\omega$. 
75. **Picture the Problem:** The geometry of the CD reader is indicated in the figure at right.

**Strategy:** Use equation 10-12 to find a relationship between the angular speed of the disk and the linear speed at the point the laser beam strikes the disk. Use the initial and final angular speeds and the time elapsed to determine the angular acceleration.

**Solution:** 1. (a) As the laser moves from the center outward, the increasing radius requires the angular speed of the disk to decrease in order to maintain the same linear speed.

2. (b) Solve equation 10-12 for $\omega$:
   
   $$\omega = \frac{v_r}{r} = \frac{1.25 \text{ m/s}}{0.0250 \text{ m}} = 50.0 \text{ rad/s}$$

3. (c) Repeat step 2 for the new radius:
   
   $$\omega = \frac{v_r}{r} = \frac{1.25 \text{ m/s}}{0.0600 \text{ m}} = 20.8 \text{ rad/s}$$

4. (d) Apply equation 10-6 directly:
   
   $$\alpha = \Delta \omega / \Delta t = \frac{\omega_2 - \omega_1}{66.5 \text{ min} \times 60 \text{ s/min}} = -7.31 \times 10^{-3} \text{ rad/s}^2$$

**Insight:** While CD players still use this variable angular speed method, most CD-ROM drives in computers use a constant angular velocity system.

78. **Picture the Problem:** The ball rolls without slipping along the horizontal table top, then falls off the edge, traveling along a parabolic arc until it hits the floor.

**Strategy:** Use the table height together with equations 4-6 to determine the time of fall. Use the number of rotations and the time elapsed to find the angular speed of the ball while it was in the air. Assuming that the angular speed was unchanged from when it was rolling on the table, equation 10-15 can be used to find the linear speed from the angular speed.

**Solution:** 1. Solve equations 4-6 for $t$:
   
   $$y = y_0 + v_{0y} t - \frac{1}{2} gt^2$$

   $$0 = y_0 + 0 - \frac{1}{2} gt^2$$

   $$t = \sqrt{2y_0/g}$$

2. Determine the angular speed $\omega$ from equation 10-3:
   
   $$\omega = \Delta \theta / \Delta t = \frac{\Delta \theta}{\sqrt{2y_0/g}}$$

3. Apply equation 10-15 directly:
   
   $$v = r\omega = r - \frac{\Delta \theta}{\sqrt{2y_0/g}} = \frac{(0.032 \text{ m})(0.37 \text{ rev} \times 2 \pi \text{ rad/rev})}{\sqrt{2(0.66 \text{ m})/(9.81 \text{ m/s}^2)}} = 0.20 \text{ m/s}$$

**Insight:** If you solve the intermediate steps you’ll find that $t = 0.37 \text{ s}$ and $\omega = 6.3 \text{ rad/s}$. If the ball is to complete one full revolution while in the air, it needs an angular speed of 17 rad/s or a linear speed of 0.55 m/s while rolling.