There are 8 different pages in this exam. Check now to see that you have all of them.

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All work and answers must be given in the spaces provided on these pages.

If you would like to get credit for having taken this exam, we need you to write your name above. Please write your name clearly.

If we catch you cheating on this exam, you will be given an “F” in this course.
Part A--Multiple Choice—40 Points Total (10 at 4 Points Each)

Only one correct answer in each question

Write your choice on the line to the left of the question number.

C 1. The density of a solid object is defined as the ratio of the mass of the object to its volume. The dimension of density is:
A) \([M]^3[L]^{-1}\]
B) \([L]/3[M]\]
C) \([M][L]^{-3}\]
D) \([M][L]^{-2}\]
E) \([M][L]^3\)

D 2. The position, \(x\), of an object is given by the equation \(x = A + Bt + Ct^2\), where \(t\) refer to time. What are the dimensions of \(A\), \(B\), and \(C\)?
A) distance, distance, distance
B) distance, time, time
C) distance, time, time^2
D) distance, distance/time, distance/time^2
E) distance/time, distance/time^2, distance/time^3

D 3. The motion of a particle is described in the velocity vs. time graph shown in Figure 1. We can say its speed
A) increases.
B) decreases.
C) increases and then decreases
D) decreases and then increases
E) remains constant

A 4. A car traveling with velocity \(v\) is decelerated by a constant acceleration of magnitude \(a\). It travels a distance \(d\) before coming to rest. If both its initial velocity and magnitude of acceleration were doubled, the distance required to stop would
A) double as well
B) decrease by a factor of two
C) stay the same
D) quadruple
E) decrease by a factor of four

A 5. You drop a stone from a bridge to the river below. After this stone has traveled a distance \(d\), you drop a second stone. The distance between the two stones will always
A) increases
B) decrease
C) stays constant
D) increases at first, but then stays constant
E) decreases at first, but then stays constant
6. An airplane starts from rest and accelerates at 10.8 m/s². What is its speed at the end of a 400 m-long runway?
A) 37.0 m/s  
B) 93.0 m/s  
C) 65.7 m/s  
D) 4320 m/s  
E) 186 m/s

7. Vector $\vec{A}$ is along x-axis and vector $\vec{B}$ is along the y-axis. Which one of the following statements is correct with respect to these vectors?
A) The x-component of vector $\vec{A}$ is equal to the x-component of vector $\vec{B}$.
B) The y-component of vector $\vec{A}$ is equal to the y-component of vector $\vec{B}$.
C) The x-component of vector $\vec{A}$ is equal and opposite to the x-component of vector $\vec{B}$.
D) The y-component of vector $\vec{B}$ is equal and opposite to the y-component of vector $\vec{A}$.
E) Vector $\vec{A}$ does not have any component along the y-axis and vector $\vec{B}$ does not have any component along the x-axis.

8. You throw a ball into the air with an initial speed of 10 m/s at an angle of 60° above the horizontal. The ball returns to the level from which it was thrown in the time $T$. Referring to Figure 2, which of the plots (1, 2, or 3) best represents the speed of the ball as a function of time?
A) Plot 1. 
B) Plot 2. 
C) Plot 3.

9. Two Projectiles are launched from the same point at the same angle above horizontal. Projectile 1 reaches a maximum height twice that of projectile 2. What is the ratio of the initial speed of projectile 1 to the initial speed of projectile 2?
A) 2  
B) 1/2  
C) $\sqrt{2}$  
D) 4  
E) 1/$\sqrt{2}$

10. The horizontal and vertical components of the initial velocity of a football are 16 m/s and 20 m/s, respectively. How long does it take for the football to rise to the highest point of its trajectory?
A) 1.0 s  
B) 2.0 s  
C) 3.0 s  
D) 4.0 s  
E) 5.0 s
Part B—Short Answer—24 Points Total (8 at 3 Points Each)

A basketball player runs down the court, following the path indicated by vectors A, B, C, D, and E in the figure below. The magnitudes of these 5 vectors are $A = 8.85\, \text{m}$, $B = 10.4\, \text{m}$, $C = 8.68\, \text{m}$, $D = 7.54\, \text{m}$, and $E = 6.43\, \text{m}$. And as shown, $\theta_1 = 38.3^\circ$, $\theta_2 = 44.6^\circ$, and $\theta_3 = 57.3^\circ$. Answer the following questions:

B-1: Write down vector $A$ in term of unit vectors: $(8.85\, \text{m})\hat{y}$

B-2: Write down vector $B$ in term of unit vectors: $(8.16\, \text{m})\hat{x} + (-6.45\, \text{m})\hat{y}$

B-3: Write down vector $C$ in term of unit vectors: $(6.18\, \text{m})\hat{x} + (6.09\, \text{m})\hat{y}$

B-4: Write down vector $D$ in term of unit vectors: $(7.54\, \text{m})\hat{x}$

B-5: Write down vector $E$ in term of unit vectors: $(-3.47\, \text{m})\hat{x} + (-5.41\, \text{m})\hat{y}$

B-6: Calculate and write down the net displacement $(\mathbf{R})$ of this player in term of unit vectors: $-(18.4\, \text{m})\hat{x} + (3.08\, \text{m})\hat{y}$

B-7: Calculate the magnitude of the net displacement $(\mathbf{R})$ of this player: $18.7\, \text{m}$

B-8: Calculate the direction of the net displacement $(\mathbf{R})$ of this player (given the angle counterclockwise relative to the right): $9.50^\circ$
Part C—Full Problems

1. Show all your work to receive full credit. A correct answer alone is worth 1 point.
2. Note that the number of points assigned to a given problem does not reflect the amount of work needed or difficulty of the problem. They reflect the importance of an idea or approach.

C-1 (18 points, broken down into 6 parts.)

A golfer hits a ball with an initial speed of 34.5 m/s at an angle of 45° above horizontal. The ball lands on a green that is 4.55 m below the ground level where the ball was struck.

C-1.1 (3 points)
How long does the ball reach the maximum height?

Set the coordinate system as shown at right

The initial velocity along x and y directions are:

\[ v_{0x} = v_0 \cos(45°) = (34.5 \text{ m/s})\cos(45°) = 24.4 \text{ m/s} \]
\[ v_{0y} = v_0 \sin(45°) = (34.5 \text{ m/s})\sin(45°) = 24.4 \text{ m/s} \]

At the maximum height, \( v_y = 0, a = -g \)

So the time used to reach the maximum height can be solved by

\[ v_y = v_{0y} - gt \Rightarrow 0 = 24.4 \text{ m/s} - (9.81 \text{ m/s}^2)t \Rightarrow t = 2.49 \text{ s} \]

C-1.2 (3 points)
Determine the maximum height of the ball above the landing level.

The height above the ground level can be calculated by:

\[ y_{max} = \frac{v_{0y}^2}{2g} - \Delta y, \text{ where } v_y = 0, v_{0y} = 24.4 \text{ m/s}, \Delta y \text{ is the height from the ground level} \]

So, \( \Delta y = \frac{v_{0y}^2}{2g} = \frac{(24.4 \text{ m/s})^2}{2*9.81 \text{ m/s}^2} = 30.3 \text{ m} \)

The maximum height above the landing level is therefore as:

\[ y_{max} = \Delta y + 4.55 \text{ m} = 30.3 \text{ m} + 4.55 \text{ m} = 34.9 \text{ m} \]

C-1.3 (2 points)
What is the speed and direction of the motion of the ball at the maximum height?

For a projectile motion, the motion along the x direction is constant velocity motion, so

\[ v_x = v_{0x} = 24.4 \text{ m/s} \]

At the maximum height, \( v_y = 0 \)

Therefore, at the maximum height, the speed of the motion is \( 24.4 \text{ m/s} \) and the direction points to positive x direction (or say “point to right”).

Copy your answers to C-1.1, C-1.2 and C-1.3 below:

For C-1.1: Time: ______2.49 s__________
For C-1.2: Maximum Height: ______34.9 m__________
For C-1.3: Speed: ______24.4 m/s__________, Direction: ______positive x direction__
C-1.4 (3 points)
How long does the ball remain in the air?

The time of the ball in the air can be calculated by

\[ y = y_0 + v_{0y} t - \frac{1}{2} gt^2, \quad \text{where } y = -4.55 \text{ m}, \quad y_0 = 0, \quad v_{0y} = 24.4 \text{ m/s} \]

Plug in the number, we can get:

\[-4.55 = 0 + (24.4 \text{ m/s})t - (1/2)(9.81 \text{ m/s}^2)t^2 \Rightarrow 4.905t^2 - (24.4 \text{ m/s})t - 4.55 = 0\]

Using quadratic equation, we can get:

\[ t_1 = 5.15 \text{ s}, \quad t_2 = -0.18 \text{ s} \text{ (meaningless)} \]

So the time of the ball remain in the air is \textbf{5.15 s}

C-1.5 (2 points)
How far has the ball traveled in the horizontal direction when it lands?

Since the motion along horizontal is constant velocity at a rate of \( v_{0x} = 24.4 \text{ m/s} \), the total fly time is \( t = 5.15 \text{ s} \), so the distance of the ball traveled along horizontal direction is:

\[ x = (v_{0x})t = (24.4 \text{ m/s})(5.15 \text{ s}) = 125.7 \text{ m} \approx 126 \text{ m}. \]

C-1.6 (4 points)
What is the speed and direction of the motion of the ball just before it lands?

At the landing point the velocity along x direction is \( v_x = v_{0x} = 24.4 \text{ m/s} \)

Along y direction, the velocity can be calculated by:

\[ v_y = v_{0y} - gt, \quad \text{where } v_{0y} = 24.4 \text{ m/s}, \quad t = 5.15 \text{ s} \]

So, \( v_y = (24.4 \text{ m/s}) - (9.81 \text{ m/s}^2)(5.15 \text{ s}) = -26.1 \text{ m/s} \) (pointing downward)

So, the speed of the ball is \( v = \sqrt{v_x^2 + v_y^2} = \sqrt{(24.4)^2 + (-26.1)^2} = 35.7 \text{ m/s} \)

The direction of the ball is \( \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-26.1}{24.4}\right) = \tan^{-1}(-1.0697) = -46.9^\circ \) (below x direction)

C-1.7 (1 points)
Write down the final velocity in term of unit vectors

The final velocity is: \( \mathbf{V} = (24.4 \text{ m/s}) \hat{x} + (-26.1 \text{ m/s}) \hat{y} \)

Copy your answers to C-1.4, C-1.5, C-1.6, and C-1.7 below:

For C-1.4: Time: \textbf{5.15 s}

For C-1.5: Distance: \textbf{126 m}

For C-1.6: Speed: \textbf{35.7 m/s} Direction: \textbf{46.9°}

For C-1.7: \textbf{(24.4 m/s)} \hat{x} + \textbf{(-26.1 m/s)} \hat{y}
C-2 (9 points, broken down into 3 parts.)

A speeder doing 60.0 km/h in a 40.0 km/h zone approaches a parked police car. The instant the speeder passes the police car, the police begin their pursuit. If the speeder accelerates from the instant he passes the police car with a constant acceleration of 1.20 m/s², and the police car accelerates with a constant acceleration of 4.80 m/s², Find:

C-2.1 (4 points)

How long does it take for the police car to catch the speeder?

The speed of the speeder is 60 km/h = 60000 m/3600 s = 16.67 m/s.
In order for the police car catches the speeder, they need to travel the same distance, i.e., \(x_p = x_s\).

For police car, \(x_p = x_{0p} + v_{0p}t + \frac{1}{2}a_p t^2\), where \(x_{0p} = 0\), \(v_{0p} = 0\), \(a_p = 4.80 \text{ m/s}^2\)
So, \(x_p = \frac{1}{2}(4.8 \text{ m/s}^2) t^2 = 2.4 t^2\)

For speeder car, \(x_s = x_{0s} + v_{0s}t + \frac{1}{2}a_s t^2\), where \(x_{0s} = 0\), \(v_{0s} = 16.67 \text{ m/s}\), \(a_s = 1.20 \text{ m/s}^2\)
So, \(x_s = (16.67 \text{ m/s})t + \frac{1}{2}(1.2 \text{ m/s}^2)t^2 = 16.67t + 0.6t^2\)

Since \(x_p = x_s\)
So, \(2.4 t^2 = 16.67t + 0.6t^2 \iff 1.8t^2 - 16.67t = 0 \iff t_1 = 0 \text{ (the instant the speeder passes the police)}, \ t_2 = 9.26 \text{ s}\)

C-2.2 (2 points)

How far have the two cars traveled in this time?

This can be solved by using both \(x_p\) and \(x_s\) equations,
\[x_p = x_{0p} + v_{0p}t + \frac{1}{2}a_p t^2 = \frac{1}{2}(4.8 \text{ m/s}^2)(9.26 \text{ s})^2 = 205.8 \text{ m} \approx 206 \text{ m}\]
\[x_s = x_{0s} + v_{0s}t + \frac{1}{2}a_s t^2 = (16.67 \text{ m/s})(9.26 \text{ s}) + \frac{1}{2}(1.2 \text{ m/s}^2)(9.26 \text{ s})^2 = 205.8 \text{ m} \approx 206 \text{ m}\]

C-2.3 (3 points)

What are the velocities of the police car and the speeder car at the instant the police catches the speeder?

For police car, \(v_p = v_{0p} + a_p t = 0 + (4.80 \text{ m/s})(9.26 \text{ s}) = 44.4 \text{ m/s} \approx 160 \text{ km/h}\)

For speeder car, \(v_s = v_{0s} + a_s t = 16.67 \text{ m/s} + (1.20 \text{ m/s})(9.26 \text{ s}) = 27.8 \text{ m/s} \approx 100 \text{ km/h}\)

Copy your answers to C-2.1, C-2.2, and C-2.3 below:

For C-2.1: Time: \(9.26 \text{ s}\)
For C-2.2: Distance: \(206 \text{ m}\)
For C-2.3: Velocity of Police Car: \(44.4 \text{ m/s}\)
Velocity of Speeder Car: \(27.8 \text{ m/s}\)
C-3. (9 points, broken down into 4 parts)
The pilot of an airplane wishes to fly due north, but there is a 75 km/h wind blowing toward east.
Find:

C-3.1 (3 points)
In what direction should the pilot head her plane if its speed relative to the air is 350 km/h? Give the angle relative to north.

The pilot should head her plane to northwest. The angle relative to north can be obtained by:

\[(350 \text{ km/h}) \sin \theta = 75 \text{ km/h} \Rightarrow \theta = \sin^{-1}(75/350) = 12.37^\circ \approx 12.4^\circ \text{ west of north}\]

C-3.2 (2 points)
Draw a vector diagram that illustrates your result in part (C-3.1).

\[v_{pg}: \text{velocity of the plane relative to the ground}\]
\[v_{pa}: \text{velocity of the plane relative to the air}\]
\[v_{ag}: \text{velocity of the air relative to the ground}\]

C-3.3 (2 points)
If the pilot decreases the air speed of the plane, but still wants to head due north, should the angle found in part (C-3.1) be increased or decreased? Explain it using several sentences.

If the plane reduces its speed but the wind velocity remains the same, the angle found in part (C3.1) should be **increased** in order for the plane to continue flying due north.

This is because \(v_{pa}\sin \theta = v_{ag}\), when \(v_{pa}\) decreases, \(v_{ag}\) is constant, \(\theta\) should be increased.

C-3.4 (2 points)
If the plane’s speed were to be reduced to 300 km/h, what is the new angle? Give the angle relative to north.

The new angle is \(\theta = \sin^{-1}(75/300) = 14.47^\circ \approx 14.5^\circ \text{ west of north}\)

**Copy your answers to C-3.1, C-3.3, and C-3.4 below:**

For C-3.1: Angle: _____12.4^\circ\text{ west of north}_____

For C-3.3: Increase or Decrease? _____ Increase _____

For C-3.4: Angle: _____14.5^\circ\text{ west of north}_____
