Coffee hour physics challenge: The Magic Hoop

Andrei Galiautdinov

Department of Physics and Astronomy, University of Georgia, Athens, GA 30602, USA
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Magic hoop’s mysterious behaviour is hand-wavingly explained.

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I. THE MEANING OF THE PHYSICAL EXPLANATION

To explain a physical phenomenon is to describe it in terms of some basic physical law(s). Here “basic” does not necessarily mean fundamental. Neither should the law be “intuitively” obvious. For example, Newton’s Second Law, as applied to rotational dynamics,

\[ \tau = \frac{dL}{dt} \] (1)

is neither fundamental nor intuitively obvious, especially when applied to various gyroscopic situations. Yet, an explanation of gyroscope’s motion given in terms of (1) is, by convention, considered acceptable, at least at the introductory physics level. We will adopt that convention in what follows.

II. STATEMENT OF THE PROBLEM

An aluminum hoop is suspended on a string and supported by a hand, as shown in Fig. 1 (left panel). If support is removed, the hoop falls. If, however, the hoop is initially spun around its center of mass, it will continue spinning in the horizontal plane, no support needed, Fig. 1 (right panel). How is that possible?

III. PAUSE

STOP HERE! Try it yourself first, carefully observe the rotational motion. If you don’t have a hoop, use a solid disk — it should also work.

FIG. 1: Experiment with a magic hoop.

OVER ⇒
IV. EXPLANATION

FIG. 2: Precession of the magic hoop.

First, if you actually did the experiment, you might have noticed that the hoop is never truly horizontal; there is always a slight tilt (as well as wobbling) relative to the vertical axis, as shown in Fig. 2 (left panel). This tilt (and the wobbling amplitude) can be made arbitrarily small by making the angular velocity of rotation sufficiently large.

We now make an assumption that the hoop is spinning sufficiently fast, so that the angular momentum, \( \vec{L} \), always points along the hoop’s axis of symmetry. In this case, as can be seen from Fig. 2 (left panel), the torque \( \vec{\tau} \) due to the tension force \( \vec{T} \) and the gravitational force \( mg \) is always horizontal and orthogonal to \( \vec{L} \) (we used the right hand rule here). Because, infinitesimally, Eq. (1) is equivalent to

\[
\vec{L} = \vec{L}_0 + \Delta \vec{L} = \vec{L}_0 + \vec{\tau} \Delta t, \quad \Delta t \to 0,
\]

the angular momentum \( \vec{L} \) maintains its magnitude but continuously changes its direction, as shown in Fig. 2 (right panel), thus “preventing” the hoop from falling. This phenomenon, known as precession, is frequently encountered in various branches of physics and astronomy.

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