Optical Rangefinding: Geometry and Gaussian Beam Physics

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A rangefinder is an optical, electrical, or acoustical instrument that is used to determine the distance of an object from the instrument. A Gaussian beam is an electromagnetic radiation beam that has transverse electric field and intensity distributions that can be approximated by Gaussian functions. Through the use of a rangefinder designed on the principles of Gaussian laser beam optics, we calculated distances from the rangefinder to various targets. We find that the rangefinder is accurate in the distance interval from 100.00 cm to 400.00 cm. However, the accuracy of the rangefinder diminishes significantly at distances below 100.00 cm and above 400.00 cm, largely due to problems with glare and intensity.

I. INTRODUCTION

Rangefinders are devices used to find the distance from the device to a target. Rangefinders vary from optical, electrical, and even acoustical. Rangefinders are primarily built on one of two basic principles: the geometric optical principle or the electro-optical principle. Geometric rangefinders have been in use for centuries. One of the earliest confirmed geometric rangefinders, the fore-staff or "ballista," was used for military ballistics applications during the Middle Ages. In the seventeenth century, optical range-finding devices based on triangulation were developed. Still widely used today, these optical triangulation devices include the stadiometric and coincidence rangefinders. Rangefinders based on the electro-optical principle did not come about until the mid-twentieth century. Electro-optical rangefinders utilize the transit time of an electromagnetic signal to estimate the range. Ultrasonic range modules, radar, and laser rangefinders are examples of range-finding methods based on electro-optical principles. Although electro-optical rangefinders are currently less accurate due to the high speed of electromagnetic waves, rangefinders based on electro-optical principles are used widely because of their ease of use and wide range of potential applications.

Rangefinders are currently used in a wide variety of fields. Rangefinders based on geometric optical principles are still used frequently in the world's militaries for ballistic, surveying, and navigation applications, but also now are used regularly in photography, the sports of hunting and golf, and in fields where surveying and navigation are commonly necessary such as forestry and archaeology. Electro-optical rangefinders have gained wide use in the military, as well as in industrial automation, in medical ultrasonography, and 3-D modeling and virtual reality simulators. Electro-optical rangefinders are also used in surveying, navigation, and sports.

Although rangefinders have a wide range of applications, most people do not have a rangefinder at home. Potential home rangefinder applications include surveying for home improvement and landscaping projects, finding space dimensions for interior design work, surveying personal real estate, and taking measurements for small construction projects. Although a personal rangefinder could be useful to many people, including first-time homeowners, home remodelers, real estate agents, home stagers, and interior designers, few people have access to a rangefinder. We postulate that the reason that most people do not have a range-finding device is price and usability. Rangefinders for personal use are over $100 and most are designed for hunting or golf use. These rangefinders are often bulky and include features specifically for hunting or golf. Bosch does manufacture a home laser range-finding device advertised as accurate to 265 feet, but it is marketed specifically to "tradesmen such as electricians, contractors, painters, masons, and builders." The price of $219 (as of 12/5/2014) for the Bosch rangefinder may be prohibitive for the average consumer. Manufacturers have noted the applications for personal and small business use of rangefinders and have created devices to fulfill that need; however current designs and price points still leave a void

for many consumers that may need something cheaper and smaller. A smartphone rangefinder would allow the average consumer to be able to facilitate the process of taking measurements.

We created a two-laser geometric optical rangefinder case for the iPhone 5 based on geometry and Gaussian optics. Using the known distance between the two lasers attached to the case and the calculated divergence angles of the two lasers, we were able to calculate the distance from the phone to the target from a photo taken with the iPhone 5 camera.

II. BASIC PHYSICS

In order to determine the most appropriate principle on which to design our rangefinder for this project, we investigated the physical principles on which the most commonly used rangefinders are based. The two basic principles of rangefinder instruments are the geometric optical principle and the electro-optical principle. Rangefinders designed using the geometric optical principle use optical devices and triangulation methods to determine a range. Triangulation is the method of determining the location of a point A by measuring the angles to point A from points B and C of a triangle. Stadiametric, and coincidence rangefinders all depend on triangulation in conjunction with the use of optical devices to determine ranges.

Stadiametric rangefinders operate on the idea that in similar triangles homologous sides are proportional. For example, when we have a right triangle with a given angle, the ratio of adjacent side length to opposite side length will be constant. Stadiametric rangefinders make use of this idea by using a reticle with marks of a known angular spacing, which can be used to find either the distance to objects of known size or the size of objects at a known distance. Both the known parameter and the angular measurement are used to derive the unknown parameter. Stadiametric rangefinders are used primarily for surveying and ballistics.6

Coincidence rangefinders are monocular. Light from the target enters the range finder through either end of the instrument, and the incident beam is reflected to the center of the optical bar by a five-sided reflecting prism located at either end. The reflected beam goes through an objective lens and is then merged with the beam of the opposing side with an ocular prism to form two images of the target. The observer views the images through the eyepiece. The beams enter the instrument at slightly different angles, resulting in an image that appears blurry. To ameliorate this blurriness, the operator adjusts a compensator to tilt the beam until the two images match. When the images match they are said to be in coincidence.7

The degree of rotation of the compensator determines the range to the target by simple triangulation. Coincidence rangefinders are used in photography, surveying, sports, and ballistics.7

Electro-optical rangefinders generally are based on the transit time of an electromagnetic signal to the target and reflect back to a sensor. Ultrasonic sensors, radar, sonar, and most laser rangefinders work on the transit-time principle. To find the distance using the transit-time principle, we use the equation:

\[ D = \frac{ct}{2} \]  

where \( D \) is the distance, \( c \) is the speed of light, and \( t \) is the transit time. Although less precise than some geometric methods, the ease-of-use and rapid results of electro-optical methods have made electro-optical rangefinding techniques prevalent in a multitude of areas including in military, medical, and industrial applications.8

After considering the various principles of optical rangefinding, we decided to build a rangefinder that operates on geometric optical principles, utilizing Gaussian beam optics. We based our design on this principle because of this principle's adaptability to a smartphone, the low cost of materials, and the knowledge and abilities of the project members.

In order to estimate the distance between the rangefinder and the targets, we used the physics of Gaussian beam optics. Generally, laser beam propagation can be approximated by assuming that the laser beam has an ideal intensity profile. The purple-blue and green lasers used in this project emit a beam with a Gaussian profile. When we assume that the laser beam has an ideal Gaussian intensity profile, we can approximate the beam's propagation. The transverse size of a Gaussian laser beam changes as it propagates. The apparent beam diameter is approximately the \( 1/e^2 \) diameter \( d \equiv 2w \), where \( w \) is the beam waist. The \( 1/e^2 \) beam diameter \( d \) varies with distance in accordance with the following equation:

\[ d(z) = \sqrt{d^2_0 + \theta^2 z^2} \]  

where \( d_0 = 2w_0 \) is the \( 1/e^2 \) beam diameter at the laser output, and \( z \) is the distance from the laser output, and \( \theta \) is the divergence angle.9

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III. METHODS

Our smartphone rangefinder case is based on geometric optical principles. The materials we used to construct the rangefinder case included one blue-purple laser with a wavelength of 405±10nm and a maximum output of less than 5mW; one green laser with a wavelength of 532±10nm and a maximum output of less than 5mW; one rugged iPhone case; dense foam; and adhesives. The lasers came in a set of three for a cost of $8.00 for all three lasers, the iPhone case was $5.35, and the dense foam cost $1.00. The total cost of the iPhone rangefinder case was $11.69, taking into account that we only used two of the three lasers in the set. We decided to use two lasers with different wavelengths to facilitate calibration and allow for more adaptability in programming methods. Before constructing the rangefinder case, we began the calibration of our device by calculating the divergence angle of each laser by measuring the diameter of each beam at 50.00cm intervals from zero to five meters away from the laser source. The divergence angle was found using a rearranged equation 2:

$$\theta = \sqrt{\frac{(dz)^2 - d_0^2}{R^2}}.$$  

Results for the divergence angles of the lasers are in the next section.

![Figure 1: Image of rangefinder device.](image1.png)

To construct the rangefinder case, we first measured the iPhone case in order to find a maximum laser placement distance. We found that the case measured 13.42cm, thus we decided on a 10.50cm distance, measured from laser center to laser center, between the two lasers. We made two laser-sized holes in the dense foam spaced 10.50cm apart, as measured from hole center to hole center. Next, we placed each laser into a hole. As part of the calibration process, we placed the apparatus 5.50m away from a wall. Then we measured the distance of the laser points on the wall and adjusted the lasers in their respective holes until the laser points on the wall were spaced 10.50cm apart (as measured by a tape measure against the wall), just as the lasers themselves were spaced 10.50cm apart in the apparatus. This calibration ensured that the lasers were not installed at an angle that would alter the spacing between the laser points with distance from the source. Any unaccounted angle in the installation of the laser would result in errors in our calculations.

![Figure 2: Screenshot of the program ImageJ in use for one picture.](image2.png)

Image analysis software ImageJ was used to process the photos. Using this software, we measured the distance from center to center of each beam, \(P\), which allowed us to set the scale for centimeters per pixel. To convert our data between image pixels and actual centimeters we needed two measurements. The spacing of the two lasers on the rangefinder case gave a fixed spacing value, \(S\), to use as a reference in our calculations. For our project, \(S = 10.50\) cm. The measured spacing, \(P\), between the purple-blue and green laser points in the target images along with \(S\) gave us the pixel per centimeter ratio, \(R\):

$$R = \frac{P}{S}.$$  

The calculated pixel per centimeter ratio allowed us to convert the diameter of each laser point in the target image to a calculated actual diameter. ImageJ was also used to measure the diameter of each laser point in the images. The calculated actual diameter was used along with the divergence angle to calculate the distance from the rangefinder to the target. To calculate the distance from the rangefinder to target we rearranged equation 2 to get:

$$z = \sqrt{\frac{(dz)^2 - d_0^2}{\theta^2}}.$$  

Where \(dz\) is the diameter at the laser source in the image, converted from pixels to centimeters, \(d_0\) is the diameter at the laser source of the laser point, and \(\theta\) is the divergence angle for the respective purple-blue and green lasers. Each laser was used to determine the distance. The average of the distances calculated from blue-purple laser and green laser data was also calculated.

In order to test the efficacy of our rangefinder case and program, we used the rangefinder case in conjunction with the iPhone to take photos at 50.00cm intervals from 50.00cm from the laser source to 550.00cm from the laser source. The tests were performed in a dimly lit room. We repeated this process twice to ensure that our results were precise.
IV. RESULTS AND DISCUSSION

Using the measured diameters at multiple distances we found the divergence angles of the two laser beams. The results are presented in figure 3.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Divergence Angle (radians)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Beam</td>
<td>0.001±0.0001</td>
</tr>
<tr>
<td>Purple-Blue Beam</td>
<td>0.003 ±0.0001</td>
</tr>
</tbody>
</table>

*Figure 3: Table showing the divergence angles of the blue-purple and green lasers*

The divergence angles of the lasers were then used alongside the beam diameter data from the rangefinder photos processed in ImageJ to calculate the rangefinder distances, shown in figure 4.

*Figure 4: Graph showing the average distance measured using the laser rangefinder’s purple and green lasers as compared to distance measured by meter stick.*

The reduced chi squared of the results was calculated to be 0.8633 with the outliers past one sigma of error removed. The values past 400.00cm had a high deviation from the actual results. This is in due part to picture resolution and the intensity of reflection to the camera being too small and distorted by ambient light.

*Figure 5: Residuals of the average calculated distance of the green and purple laser vs the measured distance. Not included is the outlier at 500cm.*

Figure 5 indicates that the calculated error is within the estimated error of 10.81cm at distances between 100.00cm and 400.00cm, but as we attempted to calculate distances past 400cm the calculated distance ceased to correlate with the actual distance. This decrease in correlation could be due to a lack of resolution and ambient noise as mentioned previously. The regular variation of the calculated error could have come from slight variations in the spacing between the two beams (faulty calibration) as well as imprecise measurements of the actual measured distance. The potential inaccuracy in meter-stick measurements was taken into account for in the calculation of error. Measurements under 100.00cm were not taken into account due to oversaturation in the rangefinding images due to glare. This oversaturation prevented plausible image processing and measurements.

V. CONCLUSION

The excellent fit between the data and the expectation value for measurements between 100.00cm and 400.00cm leads to the conclusion that the rangefinder is functional and utile for measurements in that interval. At distances less than 100.00cm, the increased error may be due to increased glare on the target and surroundings. This oversaturation could be mitigated through the use of an attenuator in future iterations of this project. At distances over 400.00cm, the increased error may be due to decreased intensity with distance. This error could be mitigated with more intense lasers, which may cause more glare at shorter distances. Further study is needed. Another source of error, may be the fluctuation in the spacing between the lasers and the laser alignment due to the qualities of dense foam and adhesives.

We have several ideas about how to improve the project. First, the professor of this class has mentioned...
access to a 3-D printer for future PHYS 3330 students. A 3-D printer would allow for the creation of a rangefinder case that could facilitate more precise and stable calibration of the rangefinder. A custom case would also be easier to use, less delicate, and less bulky. Another way to improve the rangefinder is by using the same apparatus but measuring the distance by the rate at which the pixel/cm value, $R$, changes. This allows for one measurement, the distance between the beams’ center in pixels, to be taken instead of 3 resulting in less propagation of error.

Although our rangefinder was successful for a distance interval from 100.00cm to 400.00cm, this accuracy interval is too small to replace currently available commercial rangefinders. Our rangefinder is approximately one-tenth the cost of most commercially available rangefinders, so perhaps with some of the suggested improvements, this smartphone rangefinder could be a low-cost alternative for household measurements.