Image Formation Principle

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The aim of this project was to demonstrate the image formation principle. Image formation occurs when an object is viewed or projected through a lens, and the principles describing how this happens are encapsulated in the Thin Lens, Magnification, and Lens Maker’s equations. A custom instrument was built to hold a lens, object, and camera in order to obtain a final image height created by this system. A lens was also imaged from a side view to determine the radius of curvature and verify the Lens Maker’s equation. Data gathered using both a converging and diverging lens was analyzed using derived relationships, and found to match the predicted values very well. The experimenters therefore demonstrate the efficacy of aforementioned equations in describing image formation.

I. PHYSICS

If you place some kind of non-refractive, transparent sheet in front of an object, the image appears to be the exact same size (and located in the exact same place) as the original object. However, when a lens is placed in front of an object, the observed image is altered. If a converging (or diverging) lens is placed in front of an object, the image may appear inverted as well as larger or smaller (or consequently seem farther away) than the actual object. This effect is known as the image formation principle of a lens. Three equations that are important to this principle are presented below.

\[ \frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \]  \hspace{1cm} (1)

\[ M = \frac{h_i}{h_o} = \frac{-d_i}{d_o} \]  \hspace{1cm} (2)

\[ \frac{1}{f} = \frac{n_2}{n_1} - 1 \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]  \hspace{1cm} (3)

Eq. (1) is the Lens Equation. It relates the focal length of a lens \( f \) to the distance of the object from the lens \( (d_o) \) and the distance of the formed image from the lens \( (d_i) \).

Eq. (2) is the Magnification Equation. It states that the magnification of a lens (for a certain object distance) can be described in terms of the object and image heights as well as the object and image distances. The idea for this comes from the same method used to relate triangles of similar shapes. The equation is simply the ratio of the side lengths of triangles drawn from the lens to the image and object. This formula can also be applied to a system of lenses, which we will make use of later in our experiment. For a system of two lenses, the total magnification, \( M_T \), is,

\[ M_T = \frac{h_i}{h_o} \cdot \frac{h_i'}{h_o'} = \frac{-d_i}{d_o} \cdot \frac{-d_i'}{d_o'} \]  \hspace{1cm} (4)

where image and object heights and distances for the second lens are denoted with a prime. However, in Eq. (4) \( h_o' \) is equal to \( h_i \), and \( d_i' \) can be re-written in terms of \( S \) (the distance between our lenses) and \( d_o' \).

Eq. (3) is the Lens Maker’s Equation. This equation shows how the focal length of a lens can be described in terms of its index of refraction (as well as the index of refraction of the outside medium), the radius of curvature of the outside radius of the lens, and the radius of curvature of the inside radius of the lens.

An image can be classified as either real or virtual. A real image can be projected onto a screen, because the image is formed by intersecting light rays that have passed through the lens. A virtual image, however, cannot be projected onto a screen. This is because a virtual image is not directly formed from light rays intersecting, but rather by tracing back the path of the formed rays to a point where they would intersect, as can be seen in FIG. 1.

\[ \frac{n_2}{n_1} - 1 \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

FIG. 1: A diverging lens bends light rays so that they do not intersect, but appear to have a common source.

\[ \frac{1}{f} = \frac{n_2}{n_1} - 1 \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \]

FIG. 2: A converging lens bends light rays so that they intersect to form a real image.
II. INSTRUMENT DESIGN

We designed our instrument to be able to hold an object, a lens, and a phone to allow us to study the image formation of a lens. Our instrument consists of a long, narrow track of Lego bricks with a Lego lens holder (for use with either a diverging or converging lens, both made from BK7 glass) in its center. The instrument is also designed to allow us to accurately take certain measurements by using Lego studs as a reference; the half Lego stud is defined as 4 mm (see Section III). Our instrument was split into four components: the object side, the measurement side, the lens holder, and the phone holder.

The object side was measured to be 72 half Lego studs long, the lens holder to be 4 half Lego studs long, the measurement side to be 62 half Lego studs long, and the spacing in the phone holder to be 1.5 Lego studs long.

The total dimensions of our instrument measured to be:

- **Base structure**: 144x34 half Lego studs (576x136 mm)
- **Object Side**: 72x4 half Lego studs (288x16 mm)
- **Measurement Side**: 62x4 half Lego studs (248x16 mm)
- **Lens Holder**: 4x8 half Lego studs (16x32 mm)
- **Phone Holder**: 8x10 half Lego studs (32x40 mm) with a ~1.5 half Lego stud space (~6 mm) to hold the cell phone

The lenses, purchased from Thorlabs, are made of BK-7 glass with an index of refraction of 1.5168 and a focal length of +/- 50mm (positive for the converging lens and negative for the diverging lens). The total cost of our instrument was $44.04.

III. CALIBRATION

For our experiments, we need to be able to relate measurements from our phone (in pixels) to measurements in the real world (in millimeters). This proved challenging; instead we found a way to directly relate our data on the phone to measurements in the real world without having to convert pixels to millimeters and without having to use some object of known dimensions as a reference. To do this, we examined ratios of our data. We also need to be able to account for the fact that our phone is also an imaging device and part of our system. For this we need the focal length of the lens in an iPhone 5s, which we looked up and found to be 4.12 mm. This will all be covered further in the data analysis section.

Finally, we defined a half Lego stud (or just half stud) as the distance from the edge of a Lego block to the center of the first stud, a length of 4 mm. This allowed us to record several important measurements with respect to Lego studs and later convert these values to millimeters. This method...
is an increase in precision over using a ruler and helps to reduce error.

IV. EXPERIMENTAL PROCEDURE

To test the Lens and Magnification equations for a converging and diverging lens, we first placed our lens in the lens holder and a small red Lego brick on the object side, 55 half studs (220 mm) from the lens. Then we placed our cellphone in the phone holder such that the phone’s camera was lined up with the lens and object. We took pictures at object distance increments of 4 half studs (16 mm). We defined our object as the space between two studs on our brick.

For the case of the Lens Maker’s Equation, we captured a profile view of our converging lens on camera. However, due to the shape of the diverging lens and a lack of materials, we were unable to complete this procedure for the diverging lens.

V. DATA ANALYSIS

The data analysis for this lab appears to be very straightforward (and for the Lens Maker’s Equation, it actually is), but proved to be trickier than one might initially imagine.

Image heights are determined by gradually “slicing” pixels from the raw image, until the two studs surrounding our object are just outside the range. This range of pixels is what we use for the image height.

To begin with, we considered modeling the image heights found on the camera versus an equation found from combing Eq. (1) and Eq. (2). The resulting model was:

\[ h_i = \left( \frac{f}{d_o - f} \right) h_o \]

However, our current setup does not allow us to relate the size of the image on the screen (which is in pixels) to millimeters, rendering this equation effectively useless. Instead, we came up with a clever trick to get around this. We measured how changing the object distance would affect the image height measured by the camera: we took ratios of image heights obtained at different object distances. From Eq. (5), we derive:

\[ \frac{h_i'}{h_i} = \frac{d_o - f}{d_o' - f} \]

where the prime symbol represents the height or object distance for an image that was measured one object distance increment from the previous image. By analyzing this ratio rather than the actual image height, we were able to simultaneously test the Lens and Magnification equations without having to relate the size of our image on the phone to millimeters.

Unfortunately, the data we collected does not fit our model. We realized that Eq. (6) was modeled after a single lens experiment, and this is not the actual experiment we conducted. By including our phone in the experiment, there is a second lens in the system that must be accounted for. The image captured with the camera is not the same image produced by the first lens. In fact, one might notice that the data we collected somewhat mirrors our predicted model (this would actually make sense because, shown in FIG. 6, the phone’s camera inverts the image we originally collected). But overall our results from the previous attempt are ineffective in demonstrating anything.

To account for the cell phone’s camera, we will once again combine Eq. (1) and Eq. (2) in some fashion so as to find the height of our image in terms of the focal length of our lens, the focal length of our camera lens, the original
object distance, and distance between the lens and the camera. We will once again take ratios of this equation to avoid conflicting units.

We can modify Eq. (4) to simplify it further, using the fact that \( h_i = h_o' \) and \( d_i + d_o = s \), where \( s \) is the spacing between the lens and our camera:

\[
M_R = \frac{h_o'}{h_o} = \frac{(s - d_o')}{d_o} \times \frac{d_i'}{d_i} = (s - d_o' - d_i')/d_o
\] (7)

It is important to note that in the above and following expression, primed quantities do not represent a new trial (object distance), but rather are quantities with regard to the camera lens. This notation will also be used to differentiate focal lengths. Once again we will combine Eq. (1), Eq. (2), and Eq. (7) to find the height of the image on our camera in terms of the focal length of our lens, camera lens, object distance, object height, and spacing between our lens and camera. Substitutions and subsequent algebra yield a less than elegant expression:

\[
h_o' = \frac{f \times d_o}{d_o - f} \times \frac{f'}{s - f \times d_o - f'} \times h_o
\] (8)

Therefore if the Lens and Magnification equations are true, we should be able to relate the ratio of image heights captured by our camera to the ratio of this equation for one object distance to another. And in fact by using Eq. (8) to generate models for converging and diverging lenses we notice that our new models fit the data much better (FIG. 8, 9). The model for the converging lens fits the data almost perfectly, and while the diverging lens fit is less than ideal, the data follows the overall trend of the model.

To evaluate the Lens Maker’s Equation, we uploaded a photo of our lens into python (FIG. 10). We then found the radius of curvature of the converging lens in pixels, and used the adjacent Lego blocks of known height to determine a conversion from pixels to mm. The original Lens Maker’s equation can be simplified for a plano-convex lens since there is only one radius of curvature. We therefore modify Eq. (3) by taking the limit as \( R_2 \) goes to infinity, giving:

\[
\frac{1}{f} = \left[ \frac{n_2}{n_1} - 1 \right] \left[ \frac{1}{R_1} \right]
\] (9)

with \( n_2 \) being the index of refraction of BK7 glass and \( n_1 \) the index of refraction of air. A circle of radius 313 pixels was used to fit the lens, which is equivalent to a real radius of \( \approx 26.009 \) mm. Evaluating the Lens Maker’s equation with this information yields a focal length of \( \approx 50.33 \) mm, extremely close to our expected value with a percent difference of only \( \approx 0.65\% \).
VI. DISCUSSIONS AND CONCLUSIONS

After evaluating our data and comparing it to predictive models, we determined this experiment to be an overall success. In all cases, once everything was accounted for, our models were effective at predicting the data obtained.

We found that our initial model using Eq. (6) appeared to mirror the data. If we consider the Lens and Magnification equations in tandem with the ray diagrams presented in FIG. 5, then this relationship makes sense because Eq. (6) does not account for the second lens, which is responsible for inverting the image generated by the first lens.

After taking into consideration the effect of the second lens and arriving at Eq. (8), we found that our new models fit the data much more closely. There were a few discrepancies. In the case of the converging lens, when the object distance was close to the focal length (where we change from having a virtual image to a real image) we were getting a difference of approximately 10% between the model and the data. In addition, several areas of the diverging lens data noticeably wander from the model, though differences never reach 10%. The source of these errors is unclear. General sources of error include mistakes made when measuring the image height and object distances, which likely contributed to the above discrepancies.

The Lens Maker’s equation successfully determined the focal length with no difficulty.

Overall, we successfully demonstrated that the image formation principles of a lens are well described by the Lens and Magnification equations, or more specifically, that a combination of these equations can accurately predict properties of formed images.

VII. SYSTEM EVALUATION

While the results from this experiment were very successful, there are still ways for us to improve our instrument.

Firstly, the main goal would be to increase the overall size of the instrument. While we were taking measurements, we often found locations at which measurements were impossible. There were some locations where the virtual image generated by the lens was too large to be captured in the field of view, which led to us being unable to measure the image height for that object distance. This would involve increasing the size of the lens as well as the size and width of the track. We would also choose to use lenses with larger focal lengths, particularly a converging lens with a larger focal length, in order to have more available object distances within the focal length to produce virtual images.

Secondly, it would be very beneficial to improve the quality of our lens and phone holders. Because our current holders are not very secure, the phone and lens are subject to motion, including rotating about their center of gravity. This affects the quality of the data we obtain, and eliminating this kind of error could lead to better behaved data.