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Quantum memory for superconducting qubits

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Many protocols for quantum computation require a memory element to store qubits. We discuss the speed and accuracy with which quantum states prepared in a superconducting qubit can be stored in and later retrieved from an attached high-Q resonator. The memory fidelity depends on both the qubit-resonator coupling strength and the location of the state on the Bloch sphere. Our results show that a quantum memory demonstration should be possible with existing superconducting qubit designs, which would be an important milestone in solid-state quantum information processing. Although we specifically focus on a large-area, current-biased Josesphson-junction phase qubit coupled to the dilatational mode of a piezoelectric nanoelectromechanical disk resonator, many of our results will apply to other qubit-oscillator models.

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I. INTRODUCTION

The macroscopic quantum properties of superconductors make Josephson junctions strong candidates for large-scale quantum information processing [1]. Several proposed architectures involve coupling Josephson-junction (JJ) flux, phase, or charge qubits together with LC resonators [1–9], superconducting cavities [10–13], mechanical resonators [14–16], or other types of oscillators [17–19]. Such resonator-based coupling schemes have the advantage of additional functionality resulting from the ability to tune the qubits relative to the resonator frequency, as well as to each other. Although the lowest pair of levels in a harmonic oscillator cannot be frequency selected by an external driving field, resonators are quite desirable as coupling elements because of their potential for having extremely high quality factors.

In JJ-resonator architectures, the resonators store qubit states, transfer states from one JJ to another, entangle two or more JJs, and mediate two-qubit quantum logic. In this paper we discuss the speed and accuracy with which a qubit state can be stored in a resonator and later retrieved, which depends on both the JJ-resonator coupling strength and the location of the state on the Bloch sphere. The specific architecture we consider is a large-area, current-biased JJ phase qubit coupled to a nanoelectromechanical resonator [14,16]. The Hamiltonian is similar to that of a tunable few-level atom in a single-mode electromagnetic cavity, and many of our results will apply to other qubit-oscillator models. However, the precise tradeoff between memory speed and fidelity depends on the detailed form of the JJ-resonator interaction Hamiltonian, which varies from architecture to architecture.

Transfer of simple qubit states from a JJ to a weakly coupled resonator has been considered previously in Refs. [14,16]. Here we study a complete memory operation, where the qubit is stored in the resonator and then transferred back to the JJ, for a large range of JJ-resonator coupling strengths and for a variety of qubit states. Furthermore, we show that a dramatic improvement in memory performance can be obtained by a numerical optimization procedure where the resonant interaction times and off-resonant detunings are varied to maximize the overall gate fidelity. This allows larger JJ-resonator couplings to be used, leading to faster gates and therefore more operations carried out within the available coherence time. Our results suggest that it should be possible to demonstrate a fast quantum memory using existing superconducting circuits, which would be a significant accomplishment in solid-state quantum computation.

In addition to its relevance for quantum information processing, our paper builds on recent interesting proposals to entangle a nanomechanical resonator with a Cooper-pair box [20,21]. There is considerable interest in pushing a variety of nanoelectromechanical systems (NEMS) to the quantum limit [20–24], and the memory operation described here would provide a particularly clean demonstration of quantum effects.

II. PHASE QUBIT COUPLED TO NEMS RESONATOR

The low-energy dynamics of a JJ is determined by the difference φ between the phases of the spatially uniform order parameters in the superconductors forming the junction. The Hamiltonian for the system we consider is $H=H_{\rm J}+H_{\rm res}+\delta H$, where $H_{\rm J}\equiv -E_{\rm c}(d^2/d\varphi^2)+U(\varphi)$ is the Hamiltonian for the JJ with current bias $I_{\rm b}$, with $U\equiv -E_{\rm J}(\cos\varphi+s\varphi)$ and $s\equiv I_{\rm b}/I_0$. $E_{\rm c}\equiv(2e)^2/2C$ is the charging energy, and $E_{\rm J}\equiv\hbar I_0/2e$ is the Josephson energy, with C the junction capacitance and I_0 the critical current. In the large-area JJ of interest here, $E_{\rm J}\gg E_{\rm c}$. $\omega_{\rm p0}\equiv\sqrt{2E_{\rm c}E_{\rm J}}/\hbar$ is the zero-bias plasma frequency. The lowest two eigenstates, $|0\rangle$ and $|1\rangle$, are used to make a qubit. $H_{\rm res}\equiv\hbar\omega_0 a^\dagger a$ is the Hamiltonian for the resonator, with $a^\dagger$ and a the creation and annihilation operators for dilatational-mode phonons of frequency $\omega_0<\omega_{\rm p0}$. The resonator is a piezoelectric disk sandwiched between two capacitor plates. Finally, the interaction term is $\delta H=-ig(a-a^\dagger)\varphi$, where g is a coupling constant with dimensions of energy that depends on the geometric and material properties of the resonator [14,16]. A similar interaction Hamiltonian applies to a current-biased JJ capacitively coupled to an LC circuit.

The junction Hamiltonian $H_{\rm J}$ depends on the dimensionless bias current s, which is time dependent. We expand the state of the coupled system in a basis of instantaneous eigenstates $|mn\rangle_s$ of $H_0 \equiv H_{\rm J} + H_{\rm res}$, defined by $H_0(s)|mn\rangle_s = E_{mn}(s)|mn\rangle_s$, where $|mn\rangle_s \equiv |m\rangle_{\rm J} \otimes |n\rangle_{\rm res}$. Here $|m\rangle_{\rm J}$ and $|n\rangle_{\rm res}$ are the eigenstates of the uncoupled JJ and resonator, respectively. The wave function is expanded as $|\psi(t)\rangle = \Sigma_{mn} c_{mn}(t) e^{-(i/\hbar)\int_{t_0}^t dt' E_{mn}(s)} |mn\rangle_s$. The probability amplitudes in the instantaneous interaction representation then satisfy

$$i\hbar \dot{c}_{mn} = \sum_{m'n'} \langle mn|\delta H - i\hbar \partial_t |m'n'\rangle_s \times e^{(i/\hbar)\int_{t_0}^t dt' [E_{mn}(s) - E_{m'n'}(s)]} c_{m'n'}. \tag{1}$$

All effects of dissipation and decoherence are assumed to be negligible over the time scales studied here. We will also assume that the JJ states are well approximated by harmonic oscillator eigenfunctions, which is an excellent approximation unless s is very close to unity. Transitions between instantaneous eigenstates caused by a nonadiabatic variation of s are described by the term $\langle mn|\partial/\partial t|m'n'\rangle_s = \langle mn|\partial/\partial s|m'n'\rangle_s \dot{s}$, which can be evaluated analytically in the harmonic limit [16].

III. NEMS RESONATOR AS A QUANTUM MEMORY ELEMENT

An arbitrary qubit state,

$$|\psi\rangle_{\rm J} = \alpha|0\rangle_{\rm J} + \beta|1\rangle_{\rm J}, \tag{2}$$

prepared in the JJ can be stored in the ground and one-phonon states of the resonator's dilatational mode as follows: Assuming the resonator is initially in the ground state $|0\rangle_{\rm res}$ and the JJ is detuned from the resonator, the coupled system is prepared in the initial state $(\alpha|0\rangle_{\rm J} + \beta|1\rangle_{\rm J}) \otimes |0\rangle_{\rm res} = \alpha|00\rangle + \beta|10\rangle$, with $|\alpha|^2 + |\beta|^2 = 1$. The bias current s is then adiabatically varied to tune the JJ level spacing $\Delta\epsilon = \hbar\omega_{\rm p0}(1-s^2)^{1/4}$ to $\hbar\omega_0$, reaching the resonant value $s^* \equiv \sqrt{1-(\omega_0/\omega_{\rm p0})^4}$ at time $t=0$. Neglecting any nonadiabatic corrections, the probability amplitudes in the instantaneous interaction representation at this time are $c_{mn}(0) = (\alpha\delta_{m0} + \beta\delta_{m1})\delta_{n0}$.

If the interaction strength g is small compared with $\hbar\omega_0$, the subsequent dynamics is well described by the rotating-wave approximation (RWA) of quantum optics [25]. In this approximation, and on resonance, we can write (1) as

$$\dot{c}_{0n} = \frac{g}{\hbar}\sqrt{n}\, x_{01} c_{1,n-1}$$

$$\dot{c}_{1n} = -\frac{g}{\hbar}\sqrt{n+1}\, x_{01} c_{0,n+1}, \tag{3}$$

where $x_{01} \equiv \langle 0|\varphi|1\rangle_{\rm J} = \ell^*/\sqrt{2}$ is an effective dipole moment. Here $\ell^* \equiv (2E_c/E_{\rm J})^{1/4}[1-(s^*)^2]^{-1/8}$ is the characteristic width in φ of the JJ eigenfunctions, when tuned to the resonator. Then we obtain, for $t \geq 0$,

$$c_{00}(t) = \alpha$$

$$c_{01}(t) = \beta \sin\left(\frac{\Omega t}{2}\right)$$

$$c_{10}(t) = \beta \cos\left(\frac{\Omega t}{2}\right)$$

$$c_{11}(t) = 0, \tag{4}$$

and all $c_{mn}(t)$ with $n > 1$ equal to zero. $\Omega \equiv 2gx_{01}/\hbar$ is the resonant vacuum Rabi frequency.

After a time $\Delta t = \pi/\Omega$, a π pulse, the nonvanishing probability amplitudes are $c_{00}(t) = \alpha$ and $c_{01}(t) = \beta$, corresponding to the interaction-representation state $|0\rangle_{\rm J} \otimes (\alpha|0\rangle_{\rm res} + \beta|1\rangle_{\rm res})$. The qubit state (2) has been stored in the resonator's vacuum and one-phonon states. The JJ is now adiabatically detuned from the resonator. To retrieve the stored state, we again bring the systems into resonance at time t_1. Using the stored amplitudes as initial conditions, the RWA equations now lead to (for $t \geq t_1$)

$$c_{00}(t) = \alpha$$

$$c_{01}(t) = \beta \cos\left[\frac{\Omega}{2}(t-t_1)\right]$$

$$c_{10}(t) = -\beta \sin\left[\frac{\Omega}{2}(t-t_1)\right], \tag{5}$$

the others vanishing. This time the systems are held in resonance for an interval $\Delta t = 3\pi/\Omega$, a 3π pulse, after which the original state $(\alpha|0\rangle_{\rm J} + \beta|1\rangle_{\rm J}) \otimes |0\rangle_{\rm res}$ is recovered.

The above analysis, which is based on the adiabatic approximation and the RWA, suggests that perfect memory performance can be obtained with arbitrarily fast gate times (arbitrarily large $g/\hbar\omega_0$). This is incorrect, of course, because the actual quantum memory performance is controlled by the *corrections* to these approximations, which become significant if $g/\hbar\omega_0$ is not small.

IV. QUANTUM MEMORY FIDELITY

The accuracy of a storage and retrieval operation can be characterized by the absolute value of the overlap between the intended and achieved final states, or the fidelity F of the memory operation. Accounting for the fact that the intended and actual final state have the same phase factors resulting from the time evolution of the instantaneous eigenstates, F is given by the absolute value of the inner product of the intended and achieved interaction-representation probability amplitudes. We actually report the fidelity squared,

$$F^2 = |\alpha^* c_{00}(t_{\rm f}) + \beta^* c_{10}(t_{\rm f})|^2, \tag{6}$$

which is the probability that the memory device operates correctly. The fidelity is measured at a time $t_{\rm f}$, 1 ns after the qubit has been transferred back to the JJ. The c_{mn} in (6) are obtained by numerically integrating the exact equation of motion (1).

Dissipation and decoherence are not included in our model and do not affect the F we calculate. Thus, any loss of fidelity we find is a consequence of the breakdown of the rotating-wave and adiabatic approximations. An alternative

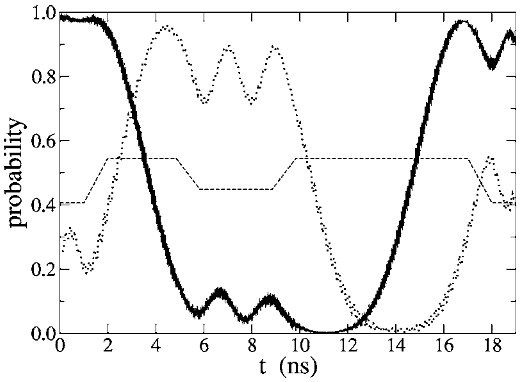

FIG. 1. Storage and retrieval of the state $2^{-1/2}(|0\rangle+|1\rangle)$. The solid curve is the overlap squared with the inital state. After about 19 ns the qubit is successfully retrieved with a squared fidelity of 91%. The dotted curve gives the occupation of the state $2^{-1/2}(|00\rangle+|01\rangle)$, in which the qubit is stored in the resonator. $g/\hbar\omega_0$ is 20%. The dashed curve is s.

approach would be to introduce phenomenological dissipation and decoherence rates for the qubit and resonator, and calculate the fidelity from the density matrix equation of motion. However, the resulting fidelity versus gate speed curve would then depend sensitively on the assumed dissipation and decoherence rates, which vary considerably from system to system. By including only the pure "gate error" contribution to F, one can immediately apply our results to a given experimental system by focusing on gate times short compared with the relevant energy and phase relaxation times.

When the JJ is weakly coupled to the resonator, with $g/\hbar\omega_0$ below a few percent, the RWA memory protocol of Sec. III works well, and qubits are stored and retrieved with high fidelity. However, such gates are *slow*. As $g/\hbar\omega_0$ is increased, making the gate faster, the fidelity becomes very poor, and it becomes necessary to deviate from the RWA protocol by numerically searching for optimum values of the resonant interaction times and off-resonant detunings. By performing this optimization for a variety of coupling strengths, we find that the π and 3π pulse times should be *shortened* to $\Delta t=[1-0.80(g/\hbar\omega_0)]\Delta t_{\mathrm{RWA}}$, where Δt_{RWA} is the RWA value.

In Fig. 1 we show the result of simulating the storage and retrieval of the qubit state $2^{-1/2}(|0\rangle+|1\rangle)$, which is on the equator of the Bloch sphere. The JJ is that of Ref. [26], with parameters $E_J=43.05$ meV and $E_c=53.33$ neV, and the resonator has a dilatational-mode frequency $\omega_0/2\pi$ of 15 GHz. The JJ and resonator are somewhat strongly coupled, with $g=0.20\,\hbar\omega_0$. The dimensionless bias current on resonance is $s^*=0.545$, and the off-resonant values were determined by optimization. Trapezoidal bias profiles with 1 ns ramps and optimized interaction times were used. The gate takes about 10 ns to complete, disregarding times during which the system is detuned, and a squared fidelity of about 91% is achieved. As stated above, the loss of fidelity comes entirely from corrections to the rotating-wave and adiabatic approximations.

A. Memory fidelity versus coupling strength

In the upper panel of Fig. 2 we plot the memory fidelity for the same qubit state $2^{-1/2}(|0\rangle+|1\rangle)$ as a function of

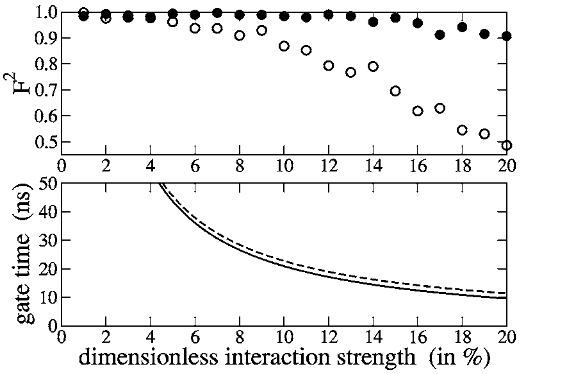

FIG. 2. (Upper panel) Memory fidelity for the equator state $2^{-1/2}(|0\rangle+|1\rangle)$ as a function of $g/\hbar\omega_0$, using both the RWA (unfilled circles) and optimized (solid circles) pulse times. (Lower panel) The time needed to store and retrieve the state, using both the RWA (dashed curve) and optimized (solid curve) pulse times.

$g/\hbar\omega_0$. As expected, the fidelity gradually decreases with increasing g. The small deviations from a strictly monotonic dependence on g are caused by oscillations in the probability amplitudes occurring when the JJ is detuned from the resonator, as in the final nanoseconds of Fig. 1. (These could be eliminated by choosing an optimal t_f for each g.) The lower panel of Fig. 2 gives the gate time as a function of $g/\hbar\omega_0$, again disregarding nonresonant evolution. These results suggest that memory fidelities better than 90% can be achieved using phase qubits and resonators with coherence times longer than a few tens of nanoseconds.

B. State dependence of memory fidelity

The memory fidelity depends not only on the strength of the JJ-resonator interaction, but also on the qubit state itself. This is because the fidelity is determined by the corrections to the adiabatic and rotating-wave approximations, and these corrections are state dependent. Using a Bloch sphere representation $\cos(\theta/2)|0\rangle+\sin(\theta/2)e^{i\phi}|1\rangle$ for the stored qubit, we show in Fig. 3 the memory fidelity along two great circles, from $|0\rangle$ to $|1\rangle$ along $\phi=0$ (left) and around the equator

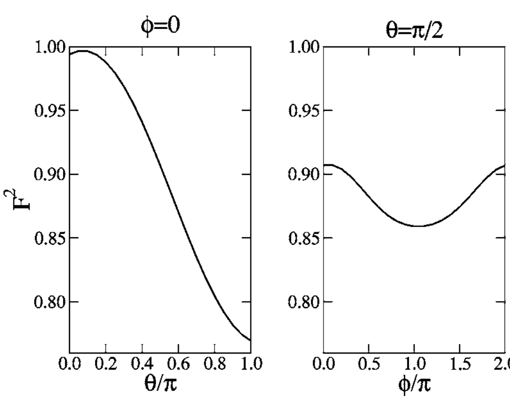

FIG. 3. State dependence of memory fidelity, for the same JJ-resonator system studied in Fig. 1, with $g/\hbar\omega_0=20\%$. (Left) Fidelity as a function of θ, along the arc $\phi=0$ on the Bloch sphere. (Right) Fidelity around the equator.

(right) starting and finishing at $2^{-1/2}(|0\rangle+|1\rangle)$.

The dependence of F on θ can be understood as follows: When $\theta=0$, the initial state of the coupled system is $|00\rangle$, because the resonator always starts in the ground state. For a weakly coupled system, $|00\rangle$ is close to the exact ground state for any s, because there are no other $|mn\rangle$ states degenerate with $|00\rangle$. In the adiabatic $ds/dt \rightarrow 0$ limit, the large component of $|00\rangle$ in the exact instantaneous ground state will remain there with unit probability, a consequence of the adiabatic theorem, leading to a high memory fidelity for the qubit state $|0\rangle$. The $|1\rangle$ state, by contrast, derives no protection from the adiabatic theorem and is subject to errors caused by the corrections to the RWA. The weaker ϕ dependence is a consequence of the form of δH, which favors equator states pointing in the $2^{-1/2}(|0\rangle+|1\rangle)$ or "positive x" direction.

V. DISCUSSION

We have explored the speed and accuracy with which a NEMS resonator can be used to store qubit states prepared in a current-biased JJ. We find that a simple optimization of resonant interaction times and off-resonant detuning leads to a tremendous improvement in gate performance compared with that resulting from the primitive RWA protocol. Finding the right balance between gate fidelity and operation time will be essential in the design of large-scale superconducting quantum computers. Overall, our simulations suggest that generic states on the Bloch sphere can be stored and retrieved in a few tens of nanoseconds with accuracies better than 90%. We expect that many of the results presented here will apply to other qubit-oscillator systems as well.

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