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Coulomb blockade in the fractional quantum Hall effect regime

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We use chiral Luttinger liquid theory to study transport through a quantum dot in the fractional quantum Hall effect regime and find rich non-Fermi-liquid tunneling characteristics. In particular, we predict a remarkable Coulomb-blockade-type energy gap that is quantized in units of the noninteracting level spacing, power-law tunneling exponents for voltages beyond threshold, and a line shape as a function of gate voltage that is dramatically different than that for a Fermi liquid. We propose experiments to use these unique spectral properties as a probe of the fractional quantum Hall effect.

Despite enormous theoretical and experimental effort during the past decade, the nature of transport in the fractional quantum Hall effect (FQHE) regime of the two-dimensional electron gas[1] remains uncertain. Although chiral Luttinger liquid (CLL) theory[2,3] has successfully predicted transport and spectral properties of sharply confined FQHE systems near the center of the $\nu=1/3$ plateau,[4] the situation at other filling factors[5] and in smooth-edged geometries[6] is poorly understood. This has motivated us to consider an alternative probe of FQHE edge states.

In a certain sense, tunneling spectra of single-branch edge states are ultimately measurements of g, the dimensionless parameter characterizing a CLL that measures the degree to which it deviates from a Fermi liquid, for which $g=1$. In particular, the zero-temperature density-of-states (DOS) of a macroscopic CLL vanishes at the Fermi energy as $\epsilon^{1/g-1}$, which is responsible for its well-known power-law tunneling characteristics. It is not surprising, and will be established below, that transport through a large quantum dot in the FQHE regime is primarily governed by the DOS of a *mesoscopic* CLL. We show here that this finite-size DOS has a remarkable low-energy structure that depends on g in an intricate manner. We therefore propose tunneling through a quantum dot in the FQHE regime as a probe of edge-state dynamics.

It has been appreciated for some time that transport through a strongly correlated FQHE droplet would be interesting in its own right, and this motivated Kinaret *et al.* to do their work on the subject.[7] Their theory, which mostly focused on the linear-response regime and on small system sizes, led to a number of proposed experiments, which have not been carried out yet. We would like to emphasize, however, that the experiments proposed by Kinaret *et al.*, and by us in the present work, although far from routine, should be possible using current nanostructure fabrication techniques.

The main difference between our work and previous work is that we have directly calculated the retarded electron propagator for a mesoscopic CLL, which has required the development of finite-size bosonization methods appropriate for the CLL.[8] As mentioned, this Green's function has a fascinating low-energy structure, which will be described below. This result has enabled us to map out a considerable portion of the low-temperature phase diagram for transport through a large quantum dot: In the $\nu=1/q$ Laughlin state with q an odd integer we predict a remarkable Coulomb-blockade-like energy gap of size $(q-1)\Delta\epsilon$, where $\Delta\epsilon$ is the noninteracting level spacing. Unlike an ordinary Coulomb blockade,[9] however, the energy gap here is quantized. Furthermore, the low-temperature tunneling current scales nonlinearly with voltage as V^q at a tunneling peak, as one might expect, but as the voltage is increased in the valley between these peaks to overcome the Coulomb blockade, the current at the threshold varies as V^{q+1}. The finite-bias line shape as a function of gate voltage depends nontrivially on q and is also dramatically different than that for a Fermi liquid.

The model we adopt here for the quantum dot system is as follows: Two macroscopic $g=1$ edge states, L and R, are weakly coupled to a mesoscopic FQHE edge state, D, in the quantum dot, by a tunneling perturbation

$$\delta H=\sum_{I=L,R}\gamma_{\rm I}\psi_{\rm I}(x_{\rm I})\,\psi_{\rm D}^{\dagger}(x_{\rm I})+\gamma_{\rm I}^{*}\psi_{\rm D}(x_{\rm I})\,\psi_{\rm I}^{\dagger}(x_{\rm I}). \tag{1}$$

The edges of the two-dimensional electron gas are assumed to be sharply confined, and the interaction short-ranged (screened by a nearby gate), so that the low-lying excitations consist of a single branch of edge magnetoplasmons with linear dispersion $\omega=v|k|$. Although we will be working at zero temperature, it is assumed that there is a small temperature present to help suppress coherence and resonant tunneling. Finally, the dot should be large enough (about 10 μm in circumference if the screening gate is 50 nm away[10]) so that the classical charging energy is smaller than the single-particle level spacing and the bulk FQHE energy gap.

What property of the electron gas is probed in a measurement of tunneling through the dot? It is known that the conductance of a simple resistive barrier measures the transmission probability of that barrier, a one-particle property, along with the single-particle Green's function of the leads, even when there is strong electron-electron interaction.[11] In contrast, transport through a quantum dot containing other electrons generally probes two-particle (and higher-order) properties of the dot, even if interactions in the leads are ignored, because the dot itself has its own internal dynamics.[12] However, for a large, weakly coupled dot, an electron can tunnel onto the dot, dissipate energy, and then tunnel incoherently through the second barrier, and in this so-called sequential tunneling limit the resistance will probe the one-particle Green's function of the dot. We estimate that in a 10 μm dot at an excitation energy of 100 mK, piezoelectric electron-phonon scattering will provide fast enough energy relaxation to permit sequential tunneling when the barrier conductances are smaller than about 10^{-2}–10^{-3} e^2/h. The opposite limit, that of coherent resonant tunneling, has been investigated previously by Kane and Fisher[13] and by Furusaki.[14]

In the sequential-tunneling limit the current from L to R is given by

$$I=\sum_N P(N)[w_{\rm LD}(N)-w_{\rm DL}(N)], \tag{2}$$

where $w_{\rm LD}$ and $w_{\rm DL}$ are transition rates to go from L to D and from D to L, given that there are N electrons in the quantum dot, and $P(N)$ is the probability that the dot has N electrons. With noninteracting leads it can be shown that

$$w_{\rm LD}(N)=2\pi|\gamma_{\rm L}|^2 N_{\rm L}(0)\Theta(V)\int_0^V d\epsilon\, N_{\rm D}(\epsilon), \tag{3}$$

where V is the electrochemical potential difference between L and D, and Θ is the unit step function. $N(\epsilon)$ is the DOS, related to the retarded electron propagator

$$G(x,t)\equiv -i\Theta(t)\langle\{\psi_\pm(x,t),\psi_\pm^\dagger(0)\}\rangle \tag{4}$$

by $N(\epsilon)\equiv-(1/\pi)\mathrm{Im}\,G(0,\epsilon)$.

The dynamics of the mesoscopic CLL is governed by the Euclidean action (here $g=1/q$ with q an odd integer)

$$S=\frac{1}{4\pi g}\int_0^L dx\int_0^\beta d\tau[\pm i\partial_\tau\phi\partial_x\phi+v(\partial_x\phi)^2], \tag{5}$$

where $\rho_\pm=\pm\partial_x\phi_\pm/2\pi$ is the charge-density fluctuation for right $(+)$ or left $(-)$ moving electrons.[3] Momentum space quantization is carried out by decomposing the chiral scalar field $\phi_\pm$ into a nonzero-mode contribution $\phi_\pm^{\rm p}$ satisfying periodic boundary conditions, and a zero-mode part $\phi_\pm^0$. The bosonized electron field is

$$\psi_\pm(x)\equiv(2\pi a)^{-1/2}e^{iq\phi_\pm(x)}e^{\pm iq\pi x/L}, \tag{6}$$

where a is a microscopic cutoff length. The grand-canonical Hamiltonian corresponding to Eq. (5) is

$$H=\frac{1}{2g}N^2\Delta\epsilon+\sum_k\Theta(\pm k)v|k|\,a_k^\dagger a_k-\mu N, \tag{7}$$

where $\Delta\epsilon\equiv 2\pi v/L$ is the noninteracting level spacing and $N\equiv\int_0^L dx\,\rho_\pm$.

At zero temperature, the bosonized electron propagator is

$$\begin{aligned}G(x,t)=&\frac{-i}{2\pi a}\Theta(t)\exp[\pm iq\pi(x\mp vt)/L]\\ &\times\langle\exp\{iq[\phi_\pm^0(x,t)-\phi_\pm^0(0)]\}\rangle\\ &\times\left(\exp\left\{\frac{1}{2}q^2[\phi_\pm^0(x,t),\phi_\pm^0(0)]\right\}e^{q^2f_\pm(x,t)}\right.\\ &\left.+\exp\left\{-\frac{1}{2}q^2[\phi_\pm^0(x,t),\phi_\pm^0(0)]\right\}e^{q^2f_\pm(-x,-t)}\right),\end{aligned}$$

where $f_\pm(x,t)\equiv\langle\phi_\pm^{\rm p}(x,t)\phi_\pm^{\rm p}(0)-(\phi_\pm^{\rm p}(0))^2\rangle$. The time evolution of the zero-mode field under the action of Eq. (7) is found to be

$$\phi_\pm^0(x,t)=\pm 2\pi N(x\mp vt)/L-g\chi+g\mu t$$

,

where $[\chi,N]=i$. Then

$$\begin{aligned}G(x,t)=&\pm\Theta(t)(i/L)^q(\pi a)^{q-1}\\ &\times\exp[\pm iq\pi(x\mp vt)/L]e^{i\mu t}\\ &\times\exp[\pm 2\pi iq\langle N\rangle(x\mp vt)/L]\\ &\times\mathrm{Im}\,\sin^{-q}[\pi(x\mp vt\pm ia)/L],\end{aligned} \tag{8}$$

where $\langle N\rangle=q^{-1}\,\mathrm{int}(\mu/\Delta\epsilon)$. Here $\mathrm{int}(x)$ denotes the integer closest to x. The Fourier transform may be written as

$$\begin{aligned}G(x,\omega)=&-\frac{i}{\pi v}\left(\frac{i\pi a}{L}\right)^{q-1}\exp[\pm 2\pi iq(\langle N\rangle+\tfrac{1}{2})x/L]\\ &\times\int_0^\infty dt\,e^{i\Omega t}\,\mathrm{Im}\left[\frac{1}{\sin^q\left(t\mp\frac{\pi x}{L}-i\frac{\pi a}{L}\right)}\right],\end{aligned} \tag{9}$$

where

$$\Omega\equiv 2\left[\frac{\omega}{\Delta\epsilon}-q\left(\langle N\rangle+\frac{1}{2}\right)+\frac{\mu}{\Delta\epsilon}\right].$$

Note that Ω depends on q both explicitly and implicitly through $\langle N\rangle$. To evaluate Eq. (9) we need integrals of the form $\int_0^\infty dt\,e^{i\Omega t}\,\mathrm{Im}\,\sin^{-q}(t+\xi-i\eta)$, which we evaluate by using the identity

$$\begin{aligned}&\int_0^{2\pi} dt\,e^{i\Omega t}\,\mathrm{Im}\,\sin^{-q}(t+\xi-i\eta)\\ &\quad=\frac{(q-2)^2-\Omega^2}{(q-1)(q-2)}\int_0^{2\pi}dt\,e^{i\Omega t}\,\mathrm{Im}\,\sin^{-(q-2)}(t+\xi-i\eta),\\ &\qquad q>2.\end{aligned} \tag{10}$$

After considerable manipulation we obtain

$$\begin{aligned}G(x,\omega)=&\frac{(i\pi a/L)^{q-1}}{(q-1)!}(1-\Omega^2)(3^2-\Omega^2)\\ &\times\cdots\times[(q-2)^2-\Omega^2]\times G_0(x,\omega),\end{aligned} \tag{11}$$

where $G_0(x,\omega)$ is the retarded propagator for the noninteracting chiral electron gas, which will be given below. The q

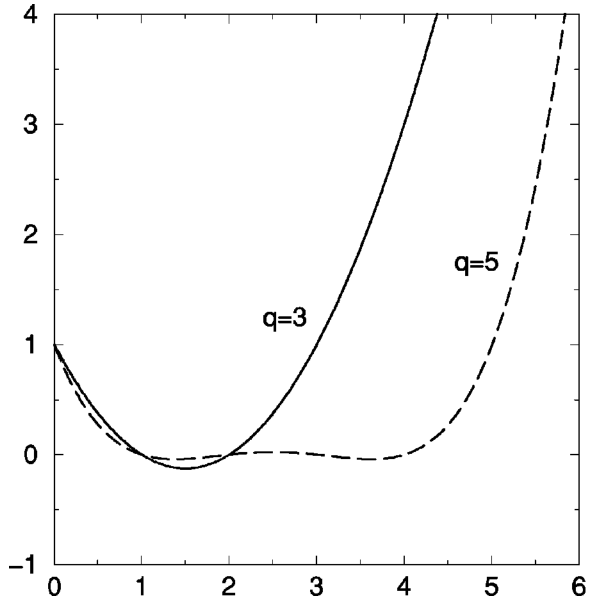

FIG. 1. Polynomial factor for the cases $q=3$ and 5, plotted as a function of $\omega/\Delta\epsilon$.

dependence of Ω is extracted by writing $\Omega=2z-q$, where $z\equiv\omega/\Delta\epsilon+\mathrm{frac}(\mu/\Delta\epsilon)$ and $\mathrm{frac}(x)\equiv x-\mathrm{int}(x)$. Finally, after using the identity (proved by induction) for q an odd integer greater than one,

$$\begin{aligned}(1-\Omega^2)(3^2-\Omega^2)\times\cdots\times[(q-2)^2-\Omega^2]\\ =[1-(2z-q)^2][3^2-(2z-q)^2]\\ \times\cdots\times[(q-2)^2-(2z-q)^2]\\ =(2i)^{q-1}\prod_{j=1}^{q-1}(z-j),\end{aligned} \tag{12}$$

we arrive at the remarkable relation

$$G(x,\omega)=G_0(x,\omega)\times\frac{1}{(q-1)!\,\epsilon_F^{q-1}}\prod_{j=1}^{q-1}(\omega-\omega_j), \tag{13}$$

where $\epsilon_F\equiv v/a$ is an effective Fermi energy and where

$$\omega_j\equiv[j-\mathrm{frac}(\mu/\Delta\epsilon)]\Delta\epsilon$$

are the noninteracting energy levels. Whereas in the $q=1$ case the propagator has poles at each of the ω_j, in the interacting case the first $q-1$ poles above μ are *removed*. This effect, which leads to a Coulomb-blockade-type energy gap, is a consequence of the first term in the Hamiltonian (7). At higher frequencies or in the large L limit where $\omega\gg\Delta\epsilon$, the additional factor becomes $\omega^{q-1}/(q-1)!\,\epsilon_F^{q-1}$, the same quantity that appears in the DOS of a macroscopic CLL.[3] The polynomial factor in Eq. (13) is plotted in Fig. 1.

In Eq. (7) we have taken the single-particle dispersion to be $\epsilon_\pm(k)=\pm vk$. The noninteracting chiral propagator is therefore

$$G_0(0,\omega)=(1/2v)\cot[\theta(\omega)/2],$$

where

$$\theta(\epsilon)=2\pi(\epsilon/\Delta\epsilon)$$

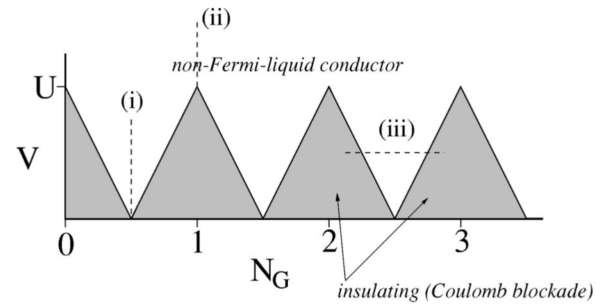

FIG. 2. Phase diagram for the tunneling current as a function of bias voltage V and gate charge N_G.

is the phase subjected to an electron of energy ϵ after going around the edge state.

Having obtained the transition rate (3) we turn to a calculation of the probability $P(N)$, which satisfies

$$\begin{aligned}\partial_t P(N)=\sum_{I=L,R}[w_{ID}(N-1)P(N-1)+w_{DI}(N+1)\\ \times P(N+1)-w_{ID}(N)P(N)-w_{DI}(N)P(N)].\end{aligned} \tag{14}$$

The steady-state solution of Eq. (14) yields the final result for the tunneling current. For the case $q=3$,[15]

$$I=2\pi|\gamma|^2N_0(0)\frac{V^2\left(V-\dfrac{4U^2}{V}\left[N_G-\left(n+\dfrac{1}{2}\right)\right]^2\right)^3}{V^2+12U^2\left[N_G-\left(n+\dfrac{1}{2}\right)\right]^2}$$

$$\text{when}\quad V>2U\left|N_G-\left(n+\frac{1}{2}\right)\right|, \tag{15}$$

and is zero otherwise (up to higher-order cotunneling terms that are neglected here). Equation (15) is valid for $n<N_G<n+1$, where the gate charge N_G is the number of positive charges induced by the gate, and for symmetric leads with DOS $N_0(\epsilon)$. n is an integer such that N_G is straddled by n and $n+1$, and $U\equiv q\Delta\epsilon$ is the quantized charging energy plus the single-particle level spacing.

The $q=3$ result (15) exhibits the novel transport properties present at all $q>1$. The Coulomb blockade boundary, shown as a solid line in Fig. 2, has the familiar diamond shape, but the scale U is now quantized in units of $\Delta\epsilon$. It can be shown that the current in a Fermi liquid would be proportional to a term of the form

$$V-\frac{4U^2}{V}\left[N_G-\left(n+\frac{1}{2}\right)\right]^2$$

alone. The additional structure present in Eq. (15) describes how the quantum dot becomes a non-Fermi-liquid conductor when threshold is exceeded. Examples of this non-Fermi-liquid behavior are shown in Fig. 2: Along path (i) the current varies as V^q, as one might naively expect, but on (ii) it varies as $(V-U)^{q+1}$. The line shape, I versus N_G, along (iii) depends nontrivially on q; for $q=3$ it varies as

$$(1-4x^2)^3/(1+12x^2),$$

which, surprisingly, is in excellent agreement with the finite-bias numerical results for just eight electrons.[7] The transport properties at other values of q can be determined from Eq. (13).

The quantization of the energy gap U in a FQHE droplet can be understood to be a consequence of the underlying Laughlin state.[16] During a sequential tunneling event, the charge of the quantum dot changes by one electronic charge e. However, the single-particle levels in a $\nu=1/q$ Laughlin state are occupied with probability $1/q$, which means that q of them are needed to accommodate the additional electron.

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