Mesoscopic Effects in the Fractional Quantum Hall Regime: Chiral Luttinger Liquid versus Fermi Liquid

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We study tunneling through an edge state formed around an antidot in the fractional quantum Hall regime using Wen's chiral Luttinger liquid theory extended to include mesoscopic effects. We identify a new regime where the Aharonov-Bohm oscillation amplitude exhibits a distinctive crossover from Luttinger liquid power-law behavior to Fermi-liquid-like behavior as the temperature is increased. Near the crossover temperature the amplitude has a pronounced maximum. This nonmonotonic behavior and novel high-temperature nonlinear phenomena that we also predict provide new ways to distinguish experimentally between Luttinger and Fermi liquids. [S0031-9007(96)01838-8]

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One of the most important outstanding questions in the study of the quantum Hall effect [1] concerns the nature of the transport in the fractional regime. It has been established that for integral filling factors, many aspects of the quantum Hall effect can be understood in terms of Halperin's edge states of the two-dimensional noninteracting electron gas [2], and a useful description of this is provided by the Büttiker-Landauer formalism [3]. However, as was shown by Laughlin [4], the fractional quantum Hall effect (FQHE) occurs because interactions lead to the formation of highly correlated incompressible states at certain filling factors. In a large class of one-dimensional systems, interactions lead to a breakdown of Fermi liquid theory and to the formation of a Luttinger liquid with bosonic low-energy excitations [5,6]. Transport in a macrosopic Luttinger liquid was studied by Kane and Fisher [7], who have shown that the conductance in the presence of a weak impurity vanishes in the zero-temperature limit, in striking contrast to a Fermi liquid. The important connection between Luttinger liquids and the FQHE was made by Wen [8], who used Chern-Simons theory to show that the edge states there should be *chiral* Luttinger liquids (CLL). Wen's proposal has stimulated a considerable effort to understand the properties of this exotic non-Fermi-liquid state [8–14].

The first observation of a CLL was reported by Milliken, Umbach, and Webb [15]. They measured the tunneling between FQHE edge states in a quantum-point-contact geometry. As the gate voltage was varied, resonance peaks in the conductance were observed that have the correct CLL temperature dependence as predicted by Moon and coworkers [10] and by Fendley, Ludwig, and Saleur [13]. In addition, Chang *et al.* [16], working with a different system, have very recently reported experimental evidence that is also in favor of CLL theory. However, recent experiments by Franklin *et al.* [17] on Aharonov-Bohm (AB) oscillations and by Maasilta and Goldman [18] on resonant tunneling in constrictions containing a quantum antidot are consistent with Fermi liquid theory. This agreement does not in itself rule out CLL theory because no CLL theory for the antidot geometry has been available. This has motivated us to provide such a theory for the experimentally realizable strong-antidot-coupling regime (a regime different from the one studied in Refs. [17,18]). Another motivation for our work is that the CLL provides a unique opportunity to study mesoscopic physics in a highly correlated electron system. Thus, our comparison of the AB effect in the Fermi and Luttinger liquids is also a novel comparison of mesoscopic effects in a noninteracting and interacting system, and we shall show that at low temperatures interactions have a dramatic effect on many quantities.

We begin by summarizing our results: Our model assumes that there is one relevant edge state at the boundaries of the device and circling the antidot. The finite size of the antidot introduces a temperature scale, $T_0 \equiv \hbar v/\pi k_B L$, where v is the edge-state Fermi velocity and L is the circumference of the antidot edge state. For example, $v = 10^6$ cm/s and $L = 1$ μm yield $T_0 \approx 25$ mK. In the strong-antidot-coupling regime, our CLL theory for filling factor $1/q$ (q odd) predicts the low-temperature ($T \ll T_0$) AB oscillation amplitude to vanish with temperature as T^{2q-2}, in striking contrast with chiral Fermi liquid theory ($q = 1$). For T near T_0, there is a pronounced maximum in the AB amplitude, also in contrast to a Fermi liquid. At high temperatures ($T \gg T_0$) we predict a crossover to a $T^{2q-1} e^{-qT/T_0}$ temperature dependence, which is qualitatively similar to chiral Fermi liquid behavior. Experiments in the strong-antidot-coupling regime should be able to distinguish between a chiral Fermi liquid and our predicted nearly Fermi liquid scaling. Finally, we predict a remarkable high-temperature nonlinear response regime, where the voltage V satisfies $V > T > T_0$, which may also be used to distinguish between chiral Fermi liquid and CLL behavior.

To study mesoscopic effects associated with FQHE edge states, we extend CLL theory to include finite-size effects. Finite-size effects in nonchiral Luttinger liquids have been addressed in Refs. [6] and [19]. We

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bosonize the electron field operators $\psi_\pm$ according to the convention $\rho_\pm = \pm\partial_x\phi_\pm/2\pi$, where $\rho_\pm$ is the normal-ordered charge density and $\phi_\pm$ is a chiral scalar field for right (+) or left (−) movers. The dynamics is governed by Wen's Euclidean action [8]

$$S_\pm = \frac{1}{4\pi g}\int_0^L dx \int_0^\beta d\tau\, \partial_x\phi_\pm(\pm i\partial_\tau\phi_\pm + v\partial_x\phi_\pm), \tag{1}$$

where $g = 1/q$ (with q odd) is the bulk filling factor. Here L is the length of a given edge state. The field theory (1) can be canonically quantized by imposing the equal-time commutation relation

$$[\phi_\pm(x), \phi_\pm(x')] = \pm i\pi g \,\mathrm{sgn}(x - x'). \tag{2}$$

We then decompose $\phi_\pm$ into a nonzero-mode contribution $\phi_\pm^{\mathrm{p}}$ satisfying periodic boundary conditions that describes the neutral excitations, and a zero-mode part $\phi_\pm^0$ that contributes to the charged excitations, $\phi_\pm = \phi_\pm^{\mathrm{p}} + \phi_\pm^0$. The nonzero-mode contribution may be expanded in a basis of Bose annihilation and creation operators in the usual fashion, $\phi_\pm^{\mathrm{p}}(x) = \sum_{k\neq 0}\theta(\pm k)\times\sqrt{2\pi g/|k|L}\,(a_k e^{ikx} + a_k^\dagger e^{-ikx})e^{-|k|a/2}$, with coefficients determined by the requirement that $\phi_\pm^{\mathrm{p}}$ itself satisfies (2) as $L\to\infty$. In a finite-size system, however, $[\phi_\pm^{\mathrm{p}}(x), \phi_\pm^{\mathrm{p}}(x')] = \pm i\pi g\,\mathrm{sgn}(x - x') \mp 2\pi i g(x - x')/L$, so we require that

$$[\phi_\pm^0(x), \phi_\pm^0(x')] = \pm 2\pi i g(x - x')/L\,. \tag{3}$$

An occupation-number expansion for$\phi_\pm^0$ is constructed from (3) and the requirement

$$\phi_\pm^0(x + L) - \phi_\pm^0(x) = \pm 2\pi N_\pm\,, \tag{4}$$

which follows from the bosonized expression for $\rho_\pm$, where $N_\pm \equiv \int_0^L dx\,\rho_\pm$ is the charge of an excited state relative to the ground state. Conditions (3) and (4) together determine $\phi_\pm^0$, up to an additive c-number constant, as $\phi_\pm^0(x) = \pm 2\pi N_\pm x/L - g\chi_\pm$, where $\chi_\pm$ is an Hermitian phase operator canonically conjugate to $N_\pm$ satisfying $[\chi_\pm, N_\pm] = i$. The normal-ordered Hamiltonian may then be written as

$$H_\pm = \frac{v}{4\pi g}\int_x (\partial_x\phi_\pm)^2 = \frac{\pi v}{gL}N_\pm^2 + \sum_k \theta(\pm k)v|k|a_k^\dagger a_k\,.$$

What are the allowed eigenvalues of $N_\pm$? The answer may be determined by bosonization: To create an electron, we need a $\pm 2\pi$ kink in $\phi_\pm$. The electron field operators can therefore be bosonized as $\psi_\pm(x) = (2\pi a)^{-1/2}e^{i[\phi_\pm(x)\pm\pi x/L]/g}$. The c-number phase factor is chosen for convenience. This implies $\psi_\pm(x + L) = \psi_\pm(x)e^{\pm i2\pi N_\pm/g}$. Periodic boundary conditions on $\psi_\pm$ then lead to the important result that the allowed eigenvalues are given by $N_\pm = ng$, which means that there exists *fractionally charged* excitations, as expected in a FQHE system.

Coupling to an additional AB flux Φ is achieved by adding $\delta\mathcal{L} = \frac{1}{c}j_\pm A$ to the Lagrangian, where $j_\pm = \pm\frac{e}{2\pi}\partial_t\phi_\pm$ is the bosonized current density and A is a vector potential. The flux couples only to the zero modes, and results in the replacement $N_\pm^2 \to (N_\pm \pm g\Phi/\Phi_0)^2$ in $H_\pm$, where $\Phi_0 \equiv hc/e$. The grand-canonical partition function of the mesoscopic edge state factorizes into a zero-mode contribution, $Z^0 = \sum_n e^{-\pi g v(n-\Phi/\Phi_0)^2/LT}$, and a flux-independent contribution from the nonzero modes [20]. Note that if the $N_\pm$ were restricted to be integral, then the partition function and the associated grand-canonical potential would be periodic functions of flux with period Φ_0/g. The fractionally charged excitations are therefore responsible for restoring the AB period to the proper value Φ_0, as is known in other contexts [21].

We begin our study of transport by performing a perturbative renormalization group (RG) analysis in the weak-antidot-coupling regime [see Fig. 1(a)]. In this case, $S = S_0 + \delta S$, where $S_0 \equiv S_{\mathrm{L}} + S_{\mathrm{R}} + S_{\mathrm{A}}$ is the sum of actions of the form (1) for the left moving, right moving, and antidot edge states, respectively, and $\delta S \equiv \sum_m \int_\tau (V_+ + V_- + \text{c.c.})$ is the weak coupling between them. Here $V_\pm(\tau) \equiv \Gamma_\pm^{(m)}e^{im\phi_\pm(x_\pm,\tau)}e^{-im\phi_A(x_\pm,\tau)}v/2\pi a$ describes the tunneling of m quasiparticles from an incident edge state into the antidot edge state at point $x_\pm$ with dimensionless amplitude $\Gamma_\pm^{(m)}$ [7]. We assume the leads, described by S_{L} and S_{R}, to be macroscopic, and we also assume for simplicity that $|\Gamma_-^{(m)}| = |\Gamma_+^{(m)}|$. We shall need the m-quasiparticle propagator $\langle e^{im\phi_\pm(x,\tau)}e^{-im\phi_\pm(0)}\rangle$ taken with respect to S_0, which, at $T = 0$ and for values of x such that $x \ll L$, is given by $[\pm ia/(x \pm iv\tau \pm ia\,\mathrm{sgn}\,\tau)]^{2\Delta}$, where $\Delta = m^2g/2$ is the scaling dimension of $e^{im\phi_\pm}$.

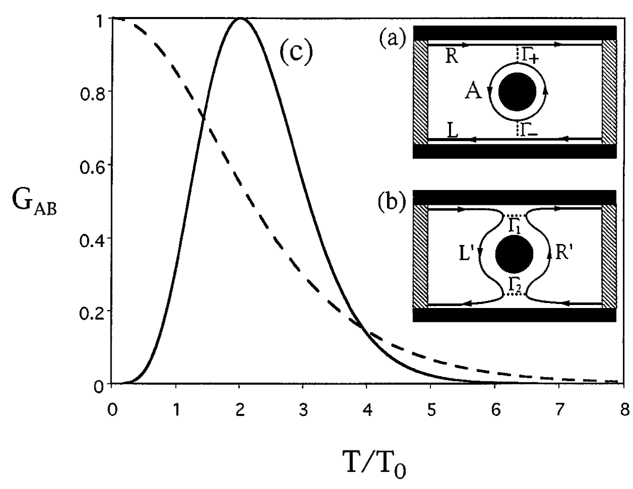

FIG. 1. (a) Aharonov-Bohm effect geometry in the weak-antidot-coupling regime. The solid lines represent edge states and the dashed lines denote weak tunneling points. (b) Edge-state configuration in the strong-antidot-coupling regime. Here the edge states are almost completely reflected. (c) Temperature dependence of G_{AB} for the cases $q = 1$ (dashed curve) and $q = 3$ (solid curve). Each curve is normalized to have unit amplitude at its maximum.

Consider now the correlation function

$$\langle V_{\pm}^{\dagger}(\tau)V_{\pm}(0)\rangle = \frac{|\Gamma_{\pm}^{(m)}|^2 v^2}{4\pi^2 a^2}\langle e^{-im\phi_{\pm}(x_{\pm},\tau)}e^{im\phi_{\pm}(x_{\pm},0)}\rangle \times \langle e^{im\phi_A(x_{\pm},\tau)}e^{-im\phi_A(x_{\pm},0)}\rangle, \quad (5)$$

which arises in a perturbative calculation of the total partition function $Z = \int \mathcal{D}\phi_L \mathcal{D}\phi_R \mathcal{D}\phi_A e^{-S}$. For Z to be invariant under a small increase in unit-cell size $a \to a' = ba$, we need $\Gamma' = b^{1-2\Delta}\Gamma$, or $d\Gamma_{\pm}^{(m)}/d\ln b = (1 - m^2 g)\Gamma_{\pm}^{(m)}$. These flow equations, which show that quasiparticle ($m = 1$) backscattering processes are relevant and electron ($m = 1/g$) backscattering is irrelevant when $g = 1/3$, were first derived by Kane and Fisher [7] using momentum-shell RG. Next consider the fourth order term $\langle V_{\pm}^{\dagger}(\tau)V_{\pm}(\tau')V_{\mp}^{\dagger}(\tau'')V_{\mp}(0)\rangle$. A Wick expansion gives *local* terms as in (5), and, in addition, *nonlocal* antidot propagators like $\langle e^{im\phi_A(x,\tau)}e^{-im\phi_A(0)}\rangle$ with $x \neq 0$. However, the nonlocal terms scale in the same way as the local terms. The Kane-Fisher flow equations are therefore valid in the antidot problem considered here.

This scaling analysis shows that off resonance [22] and at low enough temperatures the antidot will be in the strongly coupled regime shown in Fig. 1(b). Furthermore, if the antidot system starts in the strongly coupled regime, by an appropriate choice of gate voltages, it will stay in this regime because the $m = 1$ quasiparticle backscattering process (which would be relevant in the RG sense) is not allowed in this edge-state configuration and only electrons can tunnel. The strong-antidot-coupling regime therefore admits a perturbative treatment [23], to which we now turn. Details shall be given elsewhere.

The current passing between edge states L' and R', driven by their potential difference V, is defined by (restoring units) $I \equiv -e\langle \dot{N}_{L'}(t)\rangle$, where $N_{L'}$ is the charge of edge state L' as defined above. The current is now evaluated for small tunneling amplitudes $\Gamma_i (i = 1, 2)$, which for simplicity are taken to be equal apart from AB phase factors. The result is $I = -4|\Gamma|^2 \,\mathrm{Im}[X_{11}(V) + X_{12}(V)\cos(2\pi\Phi/\Phi_0)]$, where $X_{ij}(\omega = V)$ is the Fourier transform of $X_{ij}(t) \equiv -i\theta(t)\langle[B_i(t), B_j^{\dagger}(0)]\rangle$, and $B_i \equiv \psi_L(x_i)\psi_R^{\dagger}(x_i)$ is an electron tunneling operator acting at point x_i. This response function can be calculated using bosonization techniques, and the result for filling factor $1/q$ is $X_{ij}(t) = -\theta(t)(a\pi)^{2q-2}\,\mathrm{Im}[\sinh^{-q}(y_+)\sinh^{-q}(y_-)]/2L_T^{2q}$, where $L_T \equiv \beta v$ is the thermal length and $y_{\pm} \equiv \pi(x_i - x_j \pm vt \pm ia)/L_T$. Each term X_{ij} in I corresponds to a process occurring with a probability $\propto |\Gamma_i\Gamma_j|$. The *local* terms $X_{11}(= X_{22})$ therefore describe *independent* tunneling at x_1 and x_2, respectively, whereas the *nonlocal* terms $X_{12}(= X_{21})$ describe *coherent* tunneling through both points. The AB phase naturally couples only to the latter. We shall see that the local contributions behave exactly like the tunneling in a quantum point contact. The AB effect, however, is a consequence of the nonlocal terms which lead to new non-Fermi-liquid mesoscopic effects.

We have Fourier transformed $X_{ij}(t)$ exactly and find a crossover behavior in the nonlocal response functions when the thermal length L_T becomes less than $|x_i - x_j|$. The finite size of the antidot therefore provides an important new temperature scale as defined above. Note that T_0 is closely related to the energy level spacing $\Delta\epsilon \equiv 2\pi v/L$ for noninteracting electrons with linear dispersion in a ring of circumference L: $T_0 = \Delta\epsilon/2\pi^2$. The current in the strong-antidot-coupling regime can generally be written as $I = I_0 + I_{AB}\cos(2\pi\Phi/\Phi_0)$, where I_0 is the *direct* contribution resulting from the local terms and I_{AB} is the AB contribution resulting from the nonlocal terms. Thus, the AB oscillations are always sinusoidal in this regime. The widths of the resonances are temperature independent so we shall focus entirely on their amplitude. For noninteracting electrons, the Büttiker-Landauer formula or our perturbation theory with $q = 1$ shows that $I_0^{FL} = |\Gamma|^2 V/\pi$ and $I_{AB}^{FL} = 2|\Gamma|^2 T\sinh^{-1}(T/T_0)\sin(V/2\pi T_0)$. The corresponding conductances are $G_0^{FL} = |\Gamma|^2/\pi$ and $G_{AB}^{FL} = (|\Gamma|^2/\pi)(T/T_0)\sinh^{-1}(T/T_0)$.

The exact current-voltage relation for the $q = 3$ CLL is $I_0 = (|\Gamma|^2 a^4/120\pi v^6)(64\pi^4 T^4 V + 20\pi^2 T^2 V^3 + V^5)$, and

$$I_{AB} = -\frac{|\Gamma|^2 a^4 \pi^2}{v^6}\frac{T^3}{\sinh^3(T/T_0)}\left[\left\{V^2 + 4\pi^2 T^2\left[1 - 3\coth^2\left(\frac{T}{T_0}\right)\right]\right\}\sin\left(\frac{V}{2\pi T_0}\right) + 6\pi VT\coth\left(\frac{T}{T_0}\right)\cos\left(\frac{V}{2\pi T_0}\right)\right]. \quad (6)$$

In the limit $L \to 0$, I_{AB} reduces to I_0. The AB conductance is

$$G_{AB} = -\frac{2\pi^3|\Gamma|^2 a^4}{v^6}\frac{T^4}{\sinh^3(T/T_0)}\left\{3\coth\left(\frac{T}{T_0}\right) + \left(\frac{T}{T_0}\right)\left[1 - 3\coth^2\left(\frac{T}{T_0}\right)\right]\right\}, \quad (7)$$

which is shown in Fig. 1(c) along with the corresponding Fermi-liquid result.

The complete phase diagram is very rich and will be described in detail elsewhere. Here we shall summarize the transport properties for general q as a function of temperature for fixed voltage, first for $V \ll T_0$ and then for $V \gg T_0$.

Low-voltage ($V \ll T_0$) *regime.*—There are three temperature regimes here. When $T \ll V \ll T_0$, both I_0 and I_{AB} have nonlinear behavior, varying with voltage as V^{2q-1}. When the temperature exceeds V, the response becomes linear. When $V \ll T \ll T_0$, both G_0 and G_{AB} vary with temperature as

$$G \propto (T/T_F)^{2q-2}, \quad (8)$$

in striking contrast to a Fermi liquid ($q = 1$). This is the same low-temperature power-law scaling predicted [7,10,13] and observed [15] in a quantum-point-contact tunneling geometry. Here $T_F \equiv v/a$ is an effective Fermi temperature. Near $T \approx 2T_0$ for the $q = 3$ case, we find that G_{AB} displays a pronounced maximum, also in contrast to a Fermi liquid [see Fig. 1(c)]. Increasing the temperature further, however, we cross over into the $V \ll T_0 \ll T$ regime, where G_0 scales as in (8), but now

$$G_{AB} \propto (T/T_0)(T/T_F)^{2q-2}e^{-qT/T_0}. \qquad (9)$$

Thus, the AB oscillation amplitude exhibits a crossover from the well-known T^{2q-2} CLL behavior to a new scaling behavior that is much closer to a chiral Fermi liquid. Careful measurements in this experimentally accessible regime should be able to distinguish between a Fermi liquid and our predicted nearly Fermi-liquid temperature dependence.

High-voltage ($V \gg T_0$) *regime.*—Again there are three temperature regimes. At the lowest temperatures, $T \ll T_0 \ll V$, the response is nonlinear. The direct contribution varies with voltage as $I_0 \propto V^{2q-1}$. The AB current is more complicated, involving Bessel functions of the ratio $V/2\pi T_0$. As the temperature is increased further to $T_0 \ll T \ll V$, we find a crossover to a remarkable high-temperature nonlinear regime. Here, $I_0 \propto V^{2q-1}$ as before, but now $I_{AB} \propto (T/T_0)^q e^{-qT/T_0} V^{q-1} \sin(V/2\pi T_0)$. Note the additional V^{q-1} term that is not present in the corresponding Fermi-liquid result. Therefore, the nonlinear response can also be used to distinguish between Fermi liquid and CLL behavior, even at relatively high temperatures. When the temperature exceeds V, the response finally becomes linear. When $T_0 \ll V \ll T$, G_0 scales as in (8), whereas G_{AB} scales as in (9). Thus, at high temperatures the low- and high-voltage regimes behave similarly.

In conclusion, we have studied the AB effect for filling factor $1/q$ in the strong-antidot-coupling limit with CLL theory. The low-temperature linear response is similar to that in a quantum point contact. However, the AB oscillations are a mesoscopic effect and, as such, are diminished in amplitude above a crossover temperature T_0 determined by the size of the antidot. Above T_0, the temperature dependence of the AB oscillations is qualitatively similar to that in a chiral Fermi liquid [see Fig. 1(c)]. It is clear that a related crossover occurs in the weak-antidot-coupling regime as well. In addition, we have identified a new high-temperature nonlinear response regime that may also be used to distinguish between a Fermi and Luttinger liquid.

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Note added.—After this work was submitted for publication, we received a very interesting preprint by Chamon and co-workers [24], where a double point-contact arrangement that allows one to measure the charge and statistics of FQHE quasiparticles is analyzed.

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