PHYSICAL REVIEW B VOLUME 53, NUMBER 15 15 APRIL 1996-I

Current-carrying ground states in mesoscopic and macroscopic systems

Michael R. Geller
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
and Department of Physics, University of Missouri, Columbia, Missouri 65211
(Received 4 August 1995)

I extend a theorem of Bloch, which concerns the net orbital current carried by an interacting electron system in equilibrium, to include mesoscopic effects. I obtain a rigorous upper bound to the allowed ground-state current in a ring or disk, for an interacting electron system in the presence of static but otherwise arbitrary electric and magnetic fields. I also investigate the effects of spin-orbit and current-current interactions on the upper bound. Current-current interactions, caused by the magnetic field produced at a point $\mathbf{r}$ by a moving electron at $\mathbf{r}'$, are found to reduce the upper bound in a thin uniform ring by an amount that is determined by the self-inductance of the system. The upper bound is also compared with recent measurements of the persistent current in a single ring.

I. INTRODUCTION

A theorem due to Bloch holds that an interacting electron system in equilibrium carries *no* net orbital current.[1] This question originally had been motivated by early attempts, before the Bardeen-Cooper-Schrieffer theory, to explain superconductivity by proposing that electron-electron interactions lead to special current-carrying states of lower energy than the current-free states. However, it is now understood that supercurrent-carrying states are, in fact, metastable *nonequilibrium* states, which, because of their off-diagonal long-range order or wave function rigidity, have an extremely long lifetime.

The great interest over the past several years in the physics of mesoscopic systems again has made the question of allowed equilibrium currents an important one. More than ten years ago, Büttiker *et al.*[2] predicted the existence of equilibrium currents in mesoscopic normal-metal rings threaded by a magnetic flux. Recent experimental evidence[3–5] in support of this conjecture has stimulated considerable interest in these so-called persistent currents. Although a satisfactory explanation of the experiments is still lacking, the present consensus is that both electron-electron interaction and disorder effects are important.[6] A related phenomena is that of *spontaneous* orbital currents occurring in the absence of any applied magnetic field or twisted boundary conditions. Although there is no experimental evidence for this symmetry-breaking state, spontaneous orbital currents have been predicted to occur by several authors,[7–9] but to not occur by others.[10]

Given the diverse situations in which equilibrium current-carrying states may occur, it is worthwhile to revise Bloch's theorem to incorporate these mesoscopic effects. To this end, Vignale[11] has recently derived a rigorous upper bound to the persistent current in a single ring, the results being valid for both noninteracting and interacting electrons in the presence of arbitrary magnetic fields and impurity potentials. One surprising result of Vignale's analysis is that although the upper bound to the persistent current in a thin ring of uniform density and radius R vanishes as $1/R$ for large R, it does *not* vanish for a thick ring or punctured disk with the ratio $R_{\rm in}/R_{\rm out}$ of the inner radius and outer radius fixed as these radii become infinite. Although Vignale's result does not preclude the existence of a more stringent upper bound that always vanishes in the macroscopic limit, the upper bound is actually realized in calculations of the persistent current (the integrated azimuthal current density) in a two-dimensional noninteracting electron gas in a large quantum dot.[12,13]

One motivation for this work is to extend the analysis of Ref. 11 to include the effects of spin-orbit interaction, which has received considerable attention in connection with persistent currents and spontaneous currents. Spin-orbit interaction is known to lead to a topological interference effect, called the Aharonov-Casher effect,[14] which is an electromagnetic dual of the Aharonov-Bohm effect. Meir *et al.*[15] have shown that spin-orbit scattering in one-dimensional disordered rings induces an effective magnetic flux, which reduces the persistent current in a universal manner. The effect of spin-orbit interaction on mesoscopic persistent currents has been studied by several other authors, who also find reduced currents.[16–19] This has led us to question whether the upper bound on the allowed persistent current is itself reduced by spin-orbit interactions. We shall show here that this is not the case.

A second motivation for this work is to examine the influence of current-current interactions, an order v^2/c^2 relativistic effect caused by the magnetic field produced at a point $\mathbf{r}$ by a moving electron at $\mathbf{r}'$. The possibility of these magnetic interactions leading to a spontaneous current-carrying state in mesoscopic metal rings has been discussed in a remarkable paper by Wohlleben *et al.*,[7] where it is shown that, in zero field, a small ring exhibits a transition to a state with persistent current. The combined effects of spin-orbit coupling and current-current interactions has been studied recently by Choi.[9] We shall show below that current-current interactions in a thin ring reduce the upper bound on the ground-state current by an amount that is determined by the self-inductance of the system.

In this paper, we derive an upper bound to the ground-state current in an arbitrary many-electron system, including spin-orbit coupling and current-current interaction effects. To best demonstrate the modifications to Bloch's theorem from the effects of finite sample size, we restrict our analysis to zero temperature. However, our final results are also valid at

0163-1829/96/53(15)/9550(4)/$10.00

finite temperature, as may be shown by following the method of Ref. 11. Another difference between this work and Ref. 11 is that the latter derives an upper bound that is more effective (i.e., lower) in a strongly nonuniform ring. Although the upper bounds derived here are also valid in this case, they are only effective for systems with nearly uniform charge density (for example, the rings typically studied experimentally).

II. RIGOROUS UPPER BOUND

We begin by obtaining a many-electron Hamiltonian that includes current-current interactions to order v^2/c^2. In the transverse gauge, the vector potential seen by an electron at $\mathbf{r}_n$ in the presence of the other moving electrons (of charge $-e$) is

$$A^i(\mathbf{r}_n) = -(e/2c)\sum_{n'\neq n} T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})v^j_{n'}/|\mathbf{r}_n-\mathbf{r}_{n'}|, \quad (1)$$

where $\mathbf{v}_n$ is the velocity of the nth electron, and where $T^{ij}(\mathbf{r})\equiv\delta^{ij}+r^ir^j/|\mathbf{r}|^2$. This vector potential leads to a current-current interaction term in the Lagrangian of the form

$$L_{\mathrm{int}} = (e^2/2c^2)\sum_{n<n'} v^i_n T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})v^j_{n'}/|\mathbf{r}_n-\mathbf{r}_{n'}|, \quad (2)$$

which in turn leads to a current-current interaction term in the Hamiltonian of the form

$$H_{\mathrm{int}} = -(e^2/2m^2c^2)\sum_{n<n'} p^i_n \frac{T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})}{|\mathbf{r}_n-\mathbf{r}_{n'}|}p^j_{n'}, \quad (3)$$

to leading order in v^2/c^2. The complete Hamiltonian, including spin-orbit coupling, Coulomb and current-current interactions, and coupling to additional electric and magnetic fields, may be written as

$$H=\sum_n\left(\frac{\Pi_n^2}{2m}-\frac{\Pi_n^4}{8m^3c^2}+V(\mathbf{r}_n)+\frac{1}{2m^2c^2}\mathbf{S}_n\cdot[\nabla V(\mathbf{r}_n)\times\mathbf{\Pi}_n]\right)+\sum_{n<n'}\frac{e^2}{|\mathbf{r}_n-\mathbf{r}_{n'}|}-\frac{e^2}{2m^2c^2}\sum_{n<n'}\Pi^i_n\frac{T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})}{|\mathbf{r}_n-\mathbf{r}_{n'}|}\Pi^j_{n'}, \quad (4)$$

where $\mathbf{\Pi}_n\equiv\mathbf{p}_n+(e/c)\mathbf{A}(\mathbf{r}_n)$, the S^i are spin operators, and where the potentials $\mathbf{A}$ and V are time independent but otherwise arbitrary. Spin-spin interactions and the coupling of the spin degrees of freedom to the external magnetic field are not important here and shall be ignored. The velocity operator $\mathbf{v}_n\equiv[\mathbf{r}_n,H]/i\hbar$ is given by

$$v^i_n=\frac{\Pi^i_n}{m}\left(1-\frac{\Pi_n^2}{2m^2c^2}\right)+\frac{1}{2m^2c^2}[\mathbf{S}_n\times\nabla V(\mathbf{r}_n)]^i-\frac{e^2}{4m^2c^2}\sum_{n'\neq n}\left(\Pi^j_{n'}\frac{T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})}{|\mathbf{r}_n-\mathbf{r}_{n'}|}+\frac{T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})}{|\mathbf{r}_n-\mathbf{r}_{n'}|}\Pi^j_{n'}\right). \quad (5)$$

We shall consider a system of N electrons confined to a ring or disk, oriented with its axis along the z direction, and we write (4) in cylindrical coordinates $\mathbf{r}=(r,\theta,z)$. The thickness and cross-sectional shape of the system is arbitrary. The many-body ground state $\psi(\mathbf{r}_1s_1,\cdots,\mathbf{r}_Ns_N)$ satisfies $H\psi=E\psi$, where E is the ground-state energy.

Suppose that the ground state ψ carries an orbital persistent current,

$$I=-\frac{e}{4\pi}\left\langle\psi\left|\sum_n\left(\frac{\mathbf{e}_\theta(\mathbf{r}_n)}{r_n}\cdot\mathbf{v}_n+\mathbf{v}_n\cdot\frac{\mathbf{e}_\theta(\mathbf{r}_n)}{r_n}\right)\right|\psi\right\rangle, \quad (6)$$

where $\mathbf{e}_\theta(\mathbf{r})$ is an azimuthal unit vector at position $\mathbf{r}$. We may construct a *rotating* state $\psi'=U\psi$, where $U\equiv\prod_n e^{i\delta L\,\theta_n/\hbar}$, which is not necessarily an eigenstate of H, and which has a mean energy $E'\equiv\langle\psi'|H|\psi'\rangle$ given by

$$E'=E-\frac{2\pi}{e}I\,\delta L+\frac{1}{2m}\left\langle\sum_n\frac{1}{r_n^2}\right\rangle\delta L^2-\frac{1}{8m^3c^2}\left\langle\sum_n\left(\Pi_n^2\frac{1}{r_n^2}+\frac{1}{r_n^2}\Pi_n^2+4\frac{(\Pi^\theta_n)^2}{r_n^2}\right)\right\rangle\delta L^2-\frac{e^2}{4m^2c^2}\left\langle\sum_{n\neq n'}\frac{e^i_\theta(\mathbf{r}_n)T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})e^j_\theta(\mathbf{r}_{n'})}{r_n|\mathbf{r}_n-\mathbf{r}_{n'}|r_{n'}}\right\rangle\delta L^2-\frac{1}{2m^3c^2}\left\langle\sum_n\frac{\Pi^\theta_n}{r_n^3}\right\rangle\delta L^3-\frac{1}{8m^3c^2}\left\langle\sum_n\frac{1}{r_n^4}\right\rangle\delta L^4. \quad (7)$$

Here "$\langle\ \rangle$" denotes an expectation value in the original ground state ψ, $\Pi^\theta_n\equiv\mathbf{\Pi}_n\cdot\mathbf{e}_\theta(\mathbf{r}_n)$, and we have used $U^\dagger\mathbf{\Pi}_nU=\mathbf{\Pi}_n+\mathbf{e}_\theta(\mathbf{r}_n)\,\delta L/r_n$. The energy difference, $\delta E\equiv E'-E$, plotted as a function of the parameter δL, is shown in Fig. 1. For values of δL, such that $0<\delta L<\delta L^*$, where δL^* is the zero of δE defined in Fig. 1, the rotating state ψ' has a lower mean energy than ψ, so ψ cannot be the ground state.

This is the essential content of Bloch's theorem. It applies whenever there is a nonzero δL satisfying $0<\delta L<\delta L^*$. However, the smallest nonzero δL permitted by the condition that the wave function ψ' be single valued is $\delta L=\hbar$. When the relativistic corrections in (4) are neglected, $\delta L^*=\delta L_0^*$, where

$$\delta L_0^*\equiv(4\pi mI/Ne)\langle 1/r^2\rangle^{-1}. \quad (8)$$

Here,

$$\left\langle\frac{1}{r^\gamma}\right\rangle\equiv\frac{1}{N}\left\langle\sum_n\frac{1}{r_n^\gamma}\right\rangle=\frac{1}{N}\int d^3r\frac{n(\mathbf{r})}{r^\gamma}, \quad (9)$$

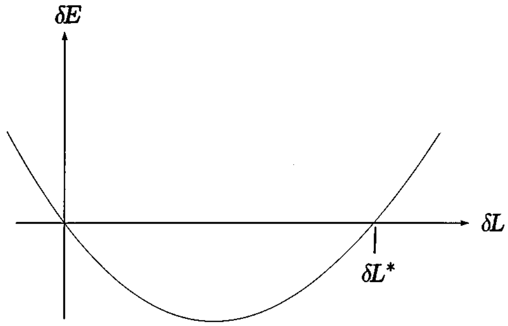

FIG. 1. Energy difference between the ground state ψ and the rotating state ψ', as a function of the imparted angular momentum δL.

where $n(\mathbf{r})$ is the ground-state number density. Therefore, Bloch's theorem applies only when $\delta L^* > \hbar$, or whenever $|I| > I^0_{\max}$, where

$$I^0_{\max} \equiv (Ne\hbar/4\pi m)\langle 1/r^2\rangle. \tag{10}$$

This is the upper bound derived in Ref. 11. When the relativistic corrections are included to leading order, δL^* is given by

$$\begin{aligned}\delta L^* = \delta L^*_0 &+ (\pi I/emc^2N^2\langle 1/r^2\rangle^2)\\ &\times\left\langle \sum_n \left(\Pi_n^2\frac{1}{r_n^2}+\frac{1}{r_n^2}\Pi_n^2+4\frac{(\Pi_n^\theta)^2}{r_n^2}\right)\right\rangle\\ &+\frac{2\pi eI}{c^2N^2\langle 1/r^2\rangle^2}\left\langle \sum_{n\neq n'} \frac{e^i_\theta(\mathbf{r}_n)T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})e^j_\theta(\mathbf{r}_{n'})}{r_n|\mathbf{r}_n-\mathbf{r}_{n'}|r_{n'}}\right\rangle\\ &+\frac{16\pi^2I^2}{e^2c^2N^3\langle 1/r^2\rangle^3}\left\langle\sum_n \frac{\Pi_n^\theta}{r_n^3}\right\rangle+\frac{16\pi^3mI^3\langle 1/r^4\rangle}{e^3c^2N^3\langle 1/r^2\rangle^4}.\end{aligned} \tag{11}$$

Bloch's theorem therefore applies whenever $|I| > I_{\max}$, where

$$I_{\max} = (1-\Lambda)I^0_{\max}, \tag{12}$$

and

$$\begin{aligned}\Lambda \equiv &\frac{1}{4m^2c^2N\langle 1/r^2\rangle}\left\langle \sum_n \left(\Pi_n^2\frac{1}{r_n^2}+\frac{1}{r_n^2}\Pi_n^2+4\frac{(\Pi_n^\theta)^2}{r_n^2}\right)\right\rangle\\ &+\frac{e^2}{2mc^2N\langle 1/r^2\rangle}\left\langle \sum_{n\neq n'} \frac{e^i_\theta(\mathbf{r}_n)T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})e^j_\theta(\mathbf{r}_{n'})}{r_n|\mathbf{r}_n-\mathbf{r}_{n'}|r_{n'}}\right\rangle\\ &+\frac{\hbar}{m^2c^2N\langle 1/r^2\rangle}\left\langle\sum_n \frac{\Pi_n^\theta}{r_n^3}\right\rangle+\frac{\hbar^2\langle 1/r^4\rangle}{4m^2c^2\langle 1/r^2\rangle}\end{aligned} \tag{13}$$

is a dimensionless *reduction factor*. States carrying orbital currents larger than $I_{\max}$ cannot be ground states of (4).

The upper bound (12) applies to interacting electron systems in the presence of static but otherwise arbitrary electric and magnetic fields, and includes the effects of spin-orbit coupling and current-current interaction. The upper bound (10) applies to noninteracting systems and also to electrons with Coulomb interaction. In particular, (10) applies to noninteracting electrons in a periodic potential, and this fact leads to a general constraint on the band structure of any one-dimensional crystal.[20]

III. UPPER BOUND FOR A THIN RING

Now consider the case of a thin ring with cross-sectional dimensions much less than the radius R of the ring. In this case,

$$I^0_{\max} = Ne\hbar/4\pi mR^2 = 2ev_F/L, \tag{14}$$

where v_F is the Fermi velocity, $L \equiv 2\pi R$ is the circumference of the ring, and

$$\begin{aligned}\Lambda \approx &\frac{3}{2m^2c^2N}\left\langle\sum_n (\Pi_n^\theta)^2\right\rangle\\ &+\frac{e^2}{2mc^2N}\left\langle \sum_{n\neq n'} \frac{e^i_\theta(\mathbf{r}_n)T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})e^j_\theta(\mathbf{r}_{n'})}{|\mathbf{r}_n-\mathbf{r}_{n'}|}\right\rangle\\ &+\frac{\hbar}{m^2c^2NR}\left\langle\sum_n \Pi_n^\theta\right\rangle+\frac{\hbar^2}{4m^2c^2R^2}.\end{aligned} \tag{15}$$

The first term in (15) is approximately equal to E_F/mc^2, where E_F is the Fermi energy, and hence this term is entirely negligible here. The magnitude of the third term may be estimated by using the approximation $\langle\Sigma_n\Pi_n^\theta\rangle \approx 4\pi mRI^0_{\max}/e$, which shows that the third term and fourth term in (15) are both of order λ_c^2/R^2, where $\lambda_c \equiv \hbar/mc$ is the Compton wavelength of the electron. These terms are, therefore, negligible here as well.

The operator in the second term of (15) may be written in second-quantized form as

$$\begin{aligned}&\sum_{n\neq n'} e^i_\theta(\mathbf{r}_n)T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})e^j_\theta(\mathbf{r}_{n'})/|\mathbf{r}_n-\mathbf{r}_{n'}|\\ &\quad=\int d^3r\; d^3r'\,F(\mathbf{r},\mathbf{r}')\,\psi^\dagger(\mathbf{r})\,\psi^\dagger(\mathbf{r}')\,\psi(\mathbf{r}')\,\psi(\mathbf{r}),\end{aligned} \tag{16}$$

where $\psi(\mathbf{r})$ and $\psi^\dagger(\mathbf{r})$ are electron field operators, and where

$$F(\mathbf{r},\mathbf{r}') \equiv e^i_\theta(\mathbf{r})T^{ij}(\mathbf{r}-\mathbf{r}')e^j_\theta(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|. \tag{17}$$

This term is a consequence of the current-current interactions. In a mesoscopic or macroscopic system, the largest contribution to the expectation value of (16) comes from the direct term $\int d^3r\; d^3r'\,F(\mathbf{r},\mathbf{r}')n(\mathbf{r})n(\mathbf{r}')$, which is normally absent in the case of Coulomb interactions in a uniform system. For a thin wire of approximately uniform density and current density, we may write the latter as I divided by the cross-sectional area N/nL,

$$\mathbf{j}(\mathbf{r}) \approx (nIL/N)\,\mathbf{e}_\theta(\mathbf{r}). \tag{18}$$

Then we have

$$\begin{aligned}&\left\langle \sum_{n\neq n'} \frac{e^i_\theta(\mathbf{r}_n)T^{ij}(\mathbf{r}_n-\mathbf{r}_{n'})e^j_\theta(\mathbf{r}_{n'})}{|\mathbf{r}_n-\mathbf{r}_{n'}|}\right\rangle\\ &\quad=\frac{N^2}{4\pi^2R^2I^2}\int d^3r\; d^3r'\,\frac{j^i(\mathbf{r})T^{ij}(\mathbf{r}-\mathbf{r}')j^j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\\ &\quad=\frac{N^2}{2\pi^2R^2I^2}\int d^3r\; d^3r'\,\frac{\mathbf{j}(\mathbf{r})\cdot\mathbf{j}_t(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|},\end{aligned} \tag{19}$$

where $\mathbf{j}_t$ is the *transverse* current density, defined as

$$\mathbf{j}_t(\mathbf{r}) \equiv (1/4\pi)\, \nabla \times \nabla \times \int d^3r'\, \mathbf{j}(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|$$

$$= \mathbf{j}(\mathbf{r}) + (1/4\pi)\, \nabla \int d^3r'\, [\nabla' \cdot \mathbf{j}(\mathbf{r}')]/|\mathbf{r}-\mathbf{r}'| . \tag{20}$$

Because the equilibrium current density satisfies $\nabla \cdot \mathbf{j} = 0$, the second term in (20) vanishes and $\mathbf{j}$ is purely transverse. The reduction factor (15) for a thin ring may therefore be written approximately as $\Lambda = 2I^0_{\max}\mathscr{L}/c\Phi_0$, where

$$\mathscr{L} \equiv (1/I^2) \int d^3r\, d^3r'\, [\mathbf{j}(\mathbf{r}) \cdot \mathbf{j}(\mathbf{r}')]/|\mathbf{r}-\mathbf{r}'| \tag{21}$$

is the classical self-inductance of the ring, and where $\Phi_0 \equiv hc/e$ is the quantum of flux. It is also useful to rewrite the reduction factor as

$$\Lambda = 2I^0_{\max}/I_c , \tag{22}$$

where

$$I_c \equiv c\Phi_0/\mathscr{L} \tag{23}$$

is the magnitude of the current needed to produce one quantum of flux. This latter form makes explicit the relative importance of the inductive effects. Therefore, the upper bound in a thin ring may be written as

$$I = (1 - 2I^0_{\max}/I_c) I^0_{\max}. \tag{24}$$

We see that the current-current interactions always *reduce* the allowed ground-state current by an amount that depends on the self-inductance of the ring. As is clear from our derivation, which treated the current-current interaction as a small perturbation, (24) is valid only when $\Lambda \ll 1$, or when $I^0_{\max} \ll I_c$. We shall evaluate (24) for realistic rings in the next section.

IV. DISCUSSION

We now evaluate the upper bound (24) for realistic thin-ring geometries. The upper bound (14) for a metal with a Fermi velocity of 10^8cm/s, a typical value, may be written as[21]

$$I^0_{\max} \approx 0.32 \;\; \mu\text{A}/L \;\; (\mu\text{m}) , \tag{25}$$

where L (μm) is the circumference of the ring in micrometers. As discussed above, the relevance of inductive effects are characterized by the ratio of $I_c \equiv c\Phi_0/\mathscr{L}$, which is the current needed to produce one quantum of flux to $I^0_{\max}$. If we measure the self-inductance in micrometers, then

$$I_c \approx 41.2 \;\; \text{mA}/\mathscr{L} \;\; (\mu\text{m}) . \tag{26}$$

The reduction factor (22) for a metal ring may then be written as

$$\Lambda \approx 1.5 \times 10^{-5} \;\; \mathscr{L}/L . \tag{27}$$

The self-inductance of a thin toroidal ring with major radius R and minor radius a (wire radius) is $\mathscr{L} = 4\pi R[\ln(8R/a) - \frac{7}{4}]$. Therefore, we see that Λ depends only on the aspect ratio R/a of the ring, and not on its circumference:

$$\Lambda \approx 3.0 \times 10^{-5}[\ln(8R/a) - \tfrac{7}{4}]. \tag{28}$$

For a thick ring, where $R/a \approx 1$, $\mathscr{L}$ is approximately equal to the size L of the ring. (Note, however, that the present analysis does not apply to this case.) For a thin ring with $R/a \approx 10$ or $R/a \approx 100$, $\mathscr{L}$ is substantially larger.

Consider, for example, a gold ring with $L \approx 12$ μm and $R/a \approx 30$, characteristic of the rings studied by Chandresakhar *et al.*,[4] where persistent currents of order $I^0_{\max}$ were measured. Then $\mathscr{L}/L \approx 7.5$, and $\Lambda \approx 10^{-4}$, a negligible reduction that is consistent with the experiments.

ACKNOWLEDGMENTS

This work was supported by the NSF through Grant No. DMR-9403908. I would like to thank Giovanni Vignale for many useful discussions on this subject. I also acknowledge the hospitality of the Institute for Theoretical Physics, Santa Barbara, where part of this work was completed under NSF Grant No. PHY89-04035.

[1] See D. Bohm, Phys. Rev. **75**, 502 (1949), and references therein.
[2] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[3] L. P. Lévy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[4] V. Chandresakhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher, and A. Kleinsasser, Phys. Rev. Lett. **67**, 3578 (1991).
[5] D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).
[6] V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 3578 (1991).
[7] D. Wohlleben, M. Esser, P. Freche, E. Zipper, and M. Szopa, Phys. Rev. Lett. **66**, 3191 (1991).
[8] M. Rasolt and F. Perrot, Phys. Rev. Lett. **69**, 2563 (1992).
[9] M. Y. Choi, Phys. Rev. Lett. **71**, 2987 (1993).
[10] D. Loss and Th. Martin, Phys. Rev. B **47**, 4619 (1993).
[11] G. Vignale, Phys. Rev. B **51**, 2612 (1995).
[12] Y. Avishai and M. Kohmoto, Phys. Rev. Lett. **71**, 279 (1993).
[13] M. R. Geller and G. Vignale, Phys. Rev. B **50**, 11 714 (1994).
[14] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319 (1984).
[15] Y. Meir, Y. Gefan, and O. Entin-Wohlman, Phys. Rev. Lett. **63**, 798 (1989).
[16] D. Loss, P. Goldbart, and A. V. Balatsky, Phys. Rev. Lett. **65**, 1655 (1990).
[17] A. V. Balatsky and B. L. Altshuler, Phys. Rev. Lett. **70**, 1678 (1993).
[18] S. Fujimoto and N. Kawakami, Phys. Rev. B **48**, 17 406 (1993).
[19] T. Z. Qian and Z. B. Su, Phys. Rev. Lett. **72**, 2311 (1994).
[20] M. R. Geller and G. Vignale, Phys. Rev. B **51**, 2616 (1995).
[21] Recall that 1 statamp $= \frac{1}{3}$ nA.