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Quantum breathing mode for electrons with $1/r^2$ interaction

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We show that the collective excitation spectrum of electrons with $1/r^2$ interaction in a parabolic quantum dot of frequency ω_0 contains a "breathing" mode of frequency 2Ω, where $\Omega^2 \equiv \omega_0^2 + \frac{1}{4}\omega_c^2$ and ω_c is the cyclotron frequency, a result first obtained by Johnson and Quiroga [Phys. Rev. Lett. **74**, 4277 (1995)].

In a recent paper,[1] Johnson and Quiroga have obtained some exact results for electrons with $1/r^2$ interaction in a two-dimensional quantum dot. A parabolic confining potential of the form $\frac{1}{2}m\omega_0^2 r^2$ is assumed, and the system is subjected to a uniform perpendicular magnetic field. In particular, they have shown that there exists a collective "breathing" mode excitation with frequency

$$\omega = 2\Omega, \tag{1}$$

where

$$\Omega^2 \equiv \omega_0^2 + \tfrac{1}{4}\omega_c^2, \tag{2}$$

and where ω_c is the cyclotron frequency. The exact spectrum of the interacting electron system therefore contains an infinite ladder of energy levels at integer multiples of $2\hbar\Omega$. However, these authors do not explain why the unphysical $1/r^2$ interaction is special, apart from the mathematical fact that it permits a separation of the many-particle Schrödinger equation in the hyperradial and hyperangular coordinates employed. Furthermore, the physical nature of the breathing mode is not fully explained.

Given the simplicity of the result (1), it is natural to ask whether there is a more direct way of obtaining it. The purpose of this paper is to point out that the quantum breathing mode excitation spectrum can also be obtained directly from the behavior of the Hamiltonian under a scale transformation, and in a manner that makes evident the special property of the inverse-square interaction for the quantum breathing mode. We shall work in the symmetric gauge and write the Hamiltonian as

$$H = T + \tfrac{1}{2}\hbar\omega_c L_z + V + U, \tag{3}$$

where

$$T \equiv \sum_n \frac{p_n^2}{2m} \tag{4}$$

is the canonical kinetic energy, with $\mathbf{p}_n$ the canonical momentum,

$$L_z \equiv \sum_n (\mathbf{r}_n \times \mathbf{p}_n)\cdot \mathbf{e}_z \tag{5}$$

is the z component of the canonical angular momentum,

$$V \equiv \sum_n \frac{1}{2} m\Omega^2 r_n^2 \tag{6}$$

is the effective field-dependent parabolic confining potential, and

$$U \equiv \sum_{n<n'} \frac{g}{|\mathbf{r}_n - \mathbf{r}_{n'}|^\alpha} \tag{7}$$

is any *power-law* electron-electron interaction.

We first note the properties of H under a scale transformation $O \rightarrow e^{i\lambda S/\hbar} O e^{-i\lambda S/\hbar}$ generated by

$$S \equiv \frac{1}{2}\sum_n (\mathbf{r}_n \cdot \mathbf{p}_n + \mathbf{p}_n \cdot \mathbf{r}_n). \tag{8}$$

This transformation performs a radial displacement of each coordinate by an amount proportional to its distance from the origin; that is, it generates a "breathing" motion. Under this transformation,

$$T \rightarrow T - 2\lambda T, \quad L_z \rightarrow L_z, \quad V \rightarrow V + 2\lambda V,$$

and

$$U \rightarrow U - \alpha\lambda U, \tag{9}$$

to first order in λ. Therefore,

$$H \rightarrow H + \frac{i}{\hbar}\lambda[S,H] = H - 2\lambda T + 2\lambda V - \alpha\lambda U. \tag{10}$$

Equation (10), however, may be regarded as an equation of motion for S. In fact, noting that

$$\frac{dV}{dt} = \Omega^2 S, \tag{11}$$

we obtain the operator equation of motion

$$\frac{d^2V}{dt^2} + (2+\alpha)\Omega^2 V = \alpha\Omega^2 (H - \tfrac{1}{2}\hbar\omega_c L_z) + (2-\alpha)\Omega^2 T, \tag{12}$$

which is the same as one would obtain classically.

The breathing mode of the corresponding classical system of point charges may be obtained from (12) by considering small oscillations about an equilibrium configuration, where the velocities are zero. Because H and L_z are constants of the motion, whereas the physical kinetic energy

$$T+\frac{\hbar\omega_c L_z}{2}+\frac{\omega_c^2 V}{4\Omega^2} \tag{13}$$

is zero to first order in the displacements, the *classical* breathing mode frequency is generally

$$\omega=\sqrt{(2+\alpha)\omega_0^2+\omega_c^2}. \tag{14}$$

For example, the classical breathing frequency of electrons with Coulomb interactions ($\alpha=1$) in a parabolic dot with no magnetic field is $\sqrt{3}\omega_0$, a result recently discovered by Peeters, Schweigert, and Bedanov.[2]

Quantum zero-point motion, however, generally modifies the breathing mode and the other classical normal modes, by shifting their frequencies and by giving them a finite width of the order of a_B/R, with a_B denoting the Bohr radius and R the radius of the droplet of charge in the dot.

An exception occurs when $\alpha=2$: In this case V becomes an *exact* quantum collective coordinate with frequency (1), independent of g and N, where N is the number of electrons. The collective coordinate V may also be separated into a center-of-mass and relative-coordinate part, $V=V_{\text{c.m.}}+V_{\text{rel}}$. For $\alpha=2$, it can be shown that each component separately satisfies a harmonic oscillator equation of motion of the form (12) with frequency 2Ω. V_{rel} is the collective coordinate corresponding to the breathing mode discussed in Ref. 1.

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[1] N. F. Johnson and L. Quiroga, Phys. Rev. Lett. **74**, 4277 (1995).

[2] F. M. Peeters, V. A. Schweigert, and V. M. Bedanov, Physica B **212**, 710 (1995). See also V. A. Schweigert and F. M. Peeters, Phys. Rev. B **51**, 7700 (1995).