A PHOTOELECTRIC ASTROMETRIC TELESCOPE USING A RONCHI RULING

ANDREW BUFFINGTON AND MICHAEL R. GELLER

Center for Astrophysics and Space Sciences (CASS), C-011, University of California—San Diego
La Jolla, California 92093

Received 1989 September 21

ABSTRACT

A new generation of photoelectric astrometric telescopes promises significant advances for both semiglobal and differential astrometry. Some of these new telescopes have a Ronchi ruling at the prime focal plane. The images move relative to the ruling, behind which photomultipliers or similar detectors view the modulated transmitted light. When several stars are viewed simultaneously, this technique exploits correlations in the atmospheric seeing to significantly improve differential astrometric precision. This article presents some general design and analysis criteria for this type of telescope and describes our particular reflector telescope. This has a 29-cm aperture and has met a 4-milli-arc-second differential precision per 5-minute observation. The system is designed for a program of improved annual parallax and solar-system object measurements.

Key words: astrometric telescopes—atmospheric seeing—differential astrometry

1. Introduction

Ground-based differential astrometric measurements have determined annual and secular motions of objects near our solar system. Global or semiglobal measurements have established fundamental stellar coordinate catalogs and tracked irregularities in Earth rotation. Trigonometric parallaxes extend the cosmic distance scale from the solar system to the closest portion of our galaxy and provide an absolute normalization for stellar-evolution calculations. However, because of technical limitations direct astrometric parallax surveys have not been able to extend far from the solar neighborhood. Only about 1000 stars are close enough, within about 20 parsecs, to have their distances determined within 10%. Extension of the astronomical distance scale beyond this has relied on a variety of less direct methods.

Traditional differential astrometric measurements have utilized carefully exposed, measured, and analyzed photographic plates. Averages of multiple exposures yield a differential angular precision of about five milli-arc-seconds. Improvements in the photographic technique, by using finer-grain emulsions and more accurate measuring machines, may reduce the above error by a factor of two to four (van Altena 1983). However, as an alternative approach, a new generation of photoelectric telescopes holds the promise of milli-arc-second precision for narrow-field differential astrometry. Monet and Dahn (1983) reported better than two milli-arc-seconds averaging over many nights with their CCD system at the prime focus of a 4-meter reflecting telescope. Gatewood (1987) and Gatewood et al. (1985a, b, 1988) report 2 to 4 milli-arc-seconds per star per night with their multichannel astrometric photometer. These new techniques are still in an early stage of development and are very promising.

In this article we discuss photoelectric astrometric telescopes which use an image-plane Ronchi ruling to encode stellar angular information in the optical band. For brevity we call these “Ronchi telescopes”. We briefly review Ronchi telescope systems here which have previously appeared in the literature and discuss some of the general design and data-analysis criteria. Then we present a particular Ronchi telescope which we have built and operated. This instrument measures the right ascension coordinate for up to 12 stars within a 0°45-wide declination band. Photomultipliers view the light transmitted through the ruling, and analysis of the responses from these provides information about both the differential and semiglobal coordinates. The system has met a differential astrometric precision of 4 milli-arc-seconds per 5-minute observation for bright stars. This instrument is different from previously described Ronchi telescopes in that it uses a single parabolic mirror to form the images and is meridian-circle mounted. It does not move during a measurement and presently has an unusually large area of sky covered by each detector.

2. Atmospheric Limits on Ground-based Astrometric Telescopes

Ground-based astrometric telescopes are limited by atmospheric refraction; this causes about an arc minute of displacement toward the zenith, when viewing at 45° from the zenith. Although a correction for this displacement can be calculated using an atmospheric model (Lang 1974), a portion of the displacement, typically a few per-
cent, varies from night to night due to changing atmospheric conditions. Moreover, dispersion spreads the stellar image along the prime vertical plane by 2% of the refraction, over the wavelength range of 0.4 to 0.6 micron. Thus, an astrometric telescope viewing away from the zenith measures apparent coordinates that depend on the spectra of the stars being measured and on the atmospheric and instrumental transmission bandpass of that particular night. At 45° from the zenith and with the above bandpass, this variability in absolute angle is about an arc second, and the differential color variability is several milli-arc-seconds. The effect on an observing program depends on the objectives: whether global, semi-global, or differential coordinates are required and whether both right ascension and declination are to be measured. Atmospheric refraction can be reduced by limiting observations to near the zenith, although for most programs the attendant small sky coverage is unacceptable. If only the right-ascension coordinate is to be measured, the telescope can integrate symmetrically around the meridian plane, since here the variability lies nearly along the declination coordinate. Finally, the effect of the color dependence of the variability can be reduced by narrowing the response bandpass of the telescope, but at the expense of a degradation of the faint magnitude limit. In principle, if several wavelength bands are separately recorded, the atmospheric refraction can be modeled and removed.

Atmospheric "seeing", the short time-scale index-of-refraction variation causing image motion, further limits ground-based astrometry. Lindegren (1980) has analyzed the seeing power spectrum; he concludes that the "pink noise" characteristic of this spectrum limits the improvement of absolute angular precision to the $\sqrt{t}$ law when the integration time exceeds about 10 minutes. This time is connected through a typical 10 to 15 m sec $^{-1}$ wind velocity, to an "outer scale", at which the turbulence becomes two-dimensional, since the third dimension becomes restricted by the 7-km atmospheric scale height.

In differential astrometry, where several stars within a small field of view are measured, the motion of the "target" star is determined relative to a coordinate frame defined by the other stars. When several stars are observed simultaneously, a substantial improvement in precision can result (Lindegren 1980). If two stars lie within an "isoplanatic patch", their motions are correlated and a higher precision is obtained for differential measurements than for the absolute angle. Lindegren predicts that an improvement proportional to the $-1/2$ power of integration time applies for this case. The size of the isoplanatic patch is related to the observing time: within a few milliseconds, only 5 to 10 arc seconds separating stars causes them to sample a substantially different air column and, thus, have different speckle patterns or centroid motion (Young 1974; Pollaine, Buffington, and Crawford 1979). For longer integration times, however, the angular size of the isoplanatic patch increases (Christian and Racine 1985), because the dominant seeing component now has a longer wavelength. The amount of improvement in differential precision over the single-star precision depends on the integration time and the angular separation of the stars being measured, but a factor of ten or more is typical.

3. Ronchi Telescopes

A Ronchi telescope has in its focal plane an optical ruling with alternating opaque and transparent stripes. The stripes are usually—but not necessarily—of equal width. Stellar images move over the ruling, either by mechanical translation of the ruling or by the rotation of the Earth. Photoelectric detectors placed behind the ruling record the modulated light and allow the determination of the line-edge crossing times. Figure 1 illustrates the basic method. Rulings are routinely made accurate to several tenths of a micron, and the substrates are stable, homogeneous glass. Exposures with these telescopes typically span several minutes to an hour, with hundreds of line edges traversed.

The first visible-wavelength Ronchi telescope was the device of Frederick et al. (1975). This telescope used a ruling rotating about its center, with a single photomultiplier to record the modulated starlight. This instrument achieved a differential angular precision of about 4 milli-arc-seconds but requires improvements in its ruling rotation system and electronics before being ready for practical application (Frederick 1985). Gatewood et al. (1980) proposed a multichannel astrometric photometer in which the ruling is translated linearly, and separate photomultipliers view the individual stars through fiber-optic light pipes. A multichannel astrometric photometer has been built for the 76-cm refractor at Allegheny Observatory and has demonstrated 2 to 4 milli-arc-second precision per star per night (Gatewood 1987; Gatewood et al. 1985a, b, 1988). Jones and York (1981, 1985) obtained a
comparable result using a reflecting telescope and an image dissector in place of photomultipliers. Although its present status is uncertain, the astrometric satellite of the European Space Agency, "HIPPARCOS", is designed to scan the sky every month for about two years with a Ronchi telescope, superimposing two star fields separated by 58° (Kovalevsky 1984). HIPPARCOS could provide unprecedented global coordinate accuracy for 100,000 stars of tenth magnitude and brighter.

4. General Design Considerations and Analysis Algorithms

Systematic errors in astrometric measurements can be introduced within the telescope itself. In a refracting telescope, changes in image position can result from thermal changes in the lens elements, not all of which can be exactly at the entrance pupil and, hence, affect differently the light from different stars. If a reflecting telescope has more than one element before the light reaches the Ronchi ruling, or if the entrance pupil is not placed at the main mirror, a similar situation exists, and thus thermal changes in the mirrors can give rise to field distortion. Changing reflectivity on sections of the mirrors can change this distortion, since the geometrical image shape retains a memory of the light-ray locations on the mirror elements. Once the light has passed through the ruling, small-angle scattering is tolerable, but systematic errors can still enter if other transmission/detection characteristics of the remaining elements placed beyond the ruling lines vary with time.

The response time series, or detector outputs as a function of time, from a Ronchi telescope consist of modulated signals as the stellar images move relative to the ruling lines. The modulation fraction for a single star in the absence of background light depends on the image size relative to the ruling linewidth. When an image is small compared with the ruling linewidth, the modulation is near 100% and the response time series consists of nearly abrupt transitions between the image fully occulted and fully transmitted. During the time that a given stellar image is not undergoing a transition between these two states, the telescope measures little angular information and thus does not take advantage of the correlated seeing effects mentioned above. At an opposite extreme, when the ruling lines are closely spaced compared with the image size, the modulation is small. Here, the image spans several open ruling lines at once, and the response time series is roughly sinusoidal.

When optimum telescope efficiency is the dominant design criterion, the width of the open ruling lines should be chosen comparable to the full width at half-maximum of the stellar images. Thus, at any time, each star in the field of view being measured has a line edge not far from the image center, probing the light distribution where it has a significant slope. Various analysis algorithms are suitable for data gathered with this ruling spacing; we have found a sinusoidal fitting algorithm to be satisfactory and a median-finding algorithm to be somewhat inferior to this. Gay and Pochet (1986a,b) have analyzed an algorithm which weights the response by its derivative. For this algorithm they conclude that an optimum width for the open ruling lines is close to the diffraction-limited full width at half-maximum of the image, with the opaque lines three times wider. Closely spaced ruling lines, together with these analysis algorithms, can admit image-shape sensitivity, which may depend in part on color, even with an all-reflecting system, due to diffraction ("chromaticity"; see Kovalevsky 1984). Here, asymmetrical aberrations in the image shift its apparent location. This in turn may limit system performance, due to the entry of systematic errors. In principle these errors can be controlled by knowing the spectrum of each star and the instrumental bandpass and by using knowledge of the image-plane position. In practice the available informa-
tion may be insufficient to retrieve full system performance.

Image-shape systematic errors can be reduced by utilizing a multielement system such as a Ritchey-Chrétien telescope design, which produces more symmetrical images than a single-mirror system, or by adoption of a Schmidt-type telescope. The latter is not easily adapted to a Ronchi ruling configuration because its image plane is curved and because a corrector may be required to reduce image size. A Ritchey-Chrétien design has a flat field but can be sensitive to image-plane distortions resulting both from thermal or temporal changes in the figure of mirror elements not at the telescope entrance pupil and from changing alignment of the various elements.

An alternative approach for overcoming sensitivity to image shape is provided for Ronchi telescopes by using the centroid of the image as the estimator of its position. Through a connection to transverse momentum conservation, the centroid propagates as a single geometrical ray beyond the final optical element, and the image location as defined by the centroid is nearly independent of the details of image shape. To see this, consider the light in a plane containing the optical axis of a single parabolic mirror. Let $\theta_i$ be the angle of a ray of reflected light relative to the optical axis. Then the momentum component of the light perpendicular to the optical axis is proportional to $\sum \sin \theta_i$, where the sum on $i$ extends over the light rays in the plane. As these rays intersect a focal plane perpendicular to the optical axis at distance $F$ from the parabolic mirror, the centroid of the light is located a distance $F \sum \tan \theta_i/I$ from the axis, where $I$ is the total number of rays. In the context of telescopes with a field of view less than one degree, such as we have here, $\sin \theta_i = \tan \theta_i$, and the centroid is thus closely tied to the total transverse momentum of the light, which propagates as a straight line as a function of $F$ and does not depend on the details of the distribution of rays. For an optical system with few elements, a large focal ratio, and not too large a field of view, this means the resulting apparent stellar field has a plate-scale change as the relative distance between the optics and ruling changes, but the resulting field is resistant to the entrance of higher-order distortions. For a similar reason, use of the centroid to define image location lessens the dependence of a multielement optical system on precise alignment of its components. With the insensitivity of the centroid to details of image shape, its use as an image-plane position estimator permits employing a single parabolic mirror as the basic element of a precision astrometric telescope, even though the images over much of the desired field of view have considerable coma.

However, the image centroid is not directly measured in a Ronchi telescope. The centroid is related to the response time series and can, in principle, be deduced from it even within the extreme of the narrow ruling lines discussed above, if the image shape is well-known over the field of view. This knowledge may not always be available and is not required with the ruling lines sufficiently wide that essentially all the light passes through only one open ruling line at some time within each line-pair passage. The response time series is then uniquely related to the image light distribution. In the context of a ruling with transparent lines this wide, and equal or wider opaque lines, Appendix A proves that the passage time of the image centroid over the midpoint of a transparent ruling line coincides with the corresponding mean of the response time series. Thus, use of the centroid analysis algorithm overcomes the systematic error potential above, although at the cost of adopting wide ruling lines with their attendant reduction of telescope efficiency.

The width chosen for the ruling lines depends on the desired level for residual systematic error. Our calculations show, for example, with a coma-dominated image 5 arc seconds long, and for systematic image-shape errors to be reduced below 10 milli-arc-seconds, that the ruling must, at some time during the passage of an image over each opaque line, occult at least 95% of the light. This requirement is governed by the combination of geometrical image size with the diffraction pattern. Apodization could reduce the diffraction wings, thus allowing a ruling linewidth not much greater than the maximum geometrical image size, usually at the edge of the field of view.

5. Apparatus

The particular telescope presented here is designed for differential astrometry to improve annual parallax measurements (Buffington 1983, 1984a,b; Buffington and Geller 1986) and for semiglobal astrometry to define coordinate frames for general relativity experiments (Buffington and Geller 1986) and for semiglobal astrometry to define coordinate frames for general relativity experiments (Buffington and Geller 1986). The instrument could also prove useful for astrometry of solar-system objects and as a navigational aid for future laser-carrying deep space probes. The telescope has a meridian-circle mount and uses a single parabolic mirror to form the images. The effect of coma-dominated, image-plane distortion is reduced by the centroid algorithm discussed above and by a symmetrical integration of data on either side of the optical axis. Restriction of the measurement to the right-ascension coordinate and having the meridian plane contain the optical axis allow further cancellation of atmospheric refractive errors. The telescope occupies its own mount and building at the Table Mountain Observatory facility of the Jet Propulsion Laboratory (Buffington 1985).

As presently configured the system measures just the right ascension for the stars. This is a modest limitation for annual parallax measurements, since the ecliptic lies at most at $23^\circ$ to the right-ascension coordinate, and thus the smallest geometrical factor associated with this restriction (cosine of $23^\circ$) is within 8% of unity. However, proper motion can have major components in both dimensions,
so the present configuration measures only half of the observables for this. The restriction is not fundamental and can be solved for this or future instruments by moving a ruling along the declination direction, by installing a "chevron" ruling with alternating sections of stripes at ±45°, or by viewing in the prime vertical away from the meridian. Of these, and away from the zenith, only the third technique is free of possible color errors due to atmospheric dispersion. However, many useful astrophysics results can be achieved by concentrating on the right-ascension coordinate, as with the configuration described here.

The design of the telescope presented here is straightforward. Nothing moves during a measurement, and a north-south alignment of the central ruling line together with the Earth’s rotation provide a ruling-modulated response for each star, whose averaged times determine the right-ascension coordinate. Possible additional systematic color errors are avoided by having the light pass through no glass prior to encountering the ruling lines. To maximize the detected photoelectrons, no filter narrows the spectral bandpass. Thus, the response bandpass is limited in the ultraviolet by atmospheric absorption and in the red by the photomultiplier (PM) response. Differential astrometry, with its attendant reduction of the correlated, seeing-induced motion described earlier, is permitted through simultaneous observation of up to 12 stars.

The Newtonian telescope design is shown in Figure 2. The field of view at prime focus is 5 cm in diameter, and coma-dominated images at the edge of this field are 64 microns in length, equivalent to 5.4 arc seconds in the sky. It employs a 32-cm diameter parabolic mirror of focal length of 245.0 cm. A principal feature of the telescope design is a maintenance of image quality at prime focus over at least a 40 K temperature variation, without observer intervention. This is accomplished by mounting the Ronchi ruling R and first diagonal mirror M2 with its support rods, in a flanged cylinder C. For thermal stability, the ruling and M2 are mounted entirely with Invar, an iron-nickel alloy having a small coefficient of thermal expansion. Cylinder C is held in place relative to mirror M1 by three Invar rods threaded into an aluminum plate behind M1. One of these rods shows at the bottom of Figure 2(a); an end protrudes through the plate at the open end of the telescope. All three ends are visible as small circles in Figure 2(b). These rods have clearance or sliding fits through the main barrel structure. The focal-plane structure C attaches to the rods at its end toward mirror M1. This attachment provides a combination in length, 91% Invar (near-zero expansion coefficient) and 9% aluminum (23. × 10⁻⁶/°K), which matches well the Pyrex M1 coefficient (2. × 10⁻⁶/°K). Thus, the focal-plane structure slides along the telescope barrel, moving a millimeter for each 20 K temperature change. Actually, the motion takes place in steps of about 25 microns, as a finite

Fig. 2—Schematic diagram of the astrometric telescope. (a) cross section through telescope barrel; (b) view down open end. i: Invar pieces; C: focal-plane structure mounted in thrust bearing; M1: primary mirror; M2, M3: diagonal mirrors; R: Ronchi ruling at prime focus; L: field lens; L1, L2: relay lenses; L3: cylindrical lens; R': reimaged focal plane; PM: photomultipliers.
force is required to overcome the static friction of the bearing; these fine increments are equivalent here to a smooth motion. The motion keeps the Ronchi ruling at the proper focal point of mirror M1, even as the Pyrex nonzero expansion coefficient changes the focal length slightly with temperature. The focal-plane structure is both counterweighted and constrained to prevent azimuthal rotation during the sliding motion. The residual rotation is observed as a rotation of the ruling on the sky. Analysis of the data to be presented in Section 6 has placed an upper limit on the standard deviation of night-to-night changes in ruling rotation of $6 \times 10^{-6}$ radians, or about one micron at the constraint. There has been no degradation of image quality seen during observatory experience, nor has there been any reason to change the focal-plane location since the original alignment at each of the two sites where the telescope has been operated.

Figure 2 also shows the secondary optics. A second diagonal mirror M3 directs the light passing through the Ronchi ruling into further optics mounted parallel to the main axis of the telescope barrel. Relay lenses L1 and L2, $f/2.5$ Aero Ektars of focal length 18.2 cm, reimaged the Ronchi ruling at location R'. Lens L, focal length 22.9 cm, is placed just behind ruling R, to image aperture A between the relay lenses. L reduces the required coverage of the relay lenses and flattens the field at R'. Cylindrical lens L3, focal length 4.3 cm, has its axis perpendicular to the plane of the figure to reimaged the telescope aperture in the right-ascension dimension, and the sky in the declination dimension, at secondary focus R'. The magnification of the relay optics is 0.98. The transmission efficiency for the present secondary optics is a low 25%. The scattering of ruling-modulated light in the relay optics causes more than one PM to respond to the passage of a given star. With fully open light pipes, this "veiling glare" is the equivalent of a star 2$''$ as bright in a neighboring open area of the ruling and are, in most cases, considerably smaller than the image size. They cause a negligible error contribution compared with the atmospheric seeing. This ruling has equal opaque and transparent lines, with widths chosen close to the maximum coma-dominated image length at both edges of the 1$''$ useful field of view for the telescope. The modulation fraction varies between 90% and 95% over the field of view. The "Ronchi frequency" is 1.4 Hz at zero declination.

The sky covered by the ruling at prime focus R, and reimaged in the declination dimension at R', is approximately 1$^\circ$2 in right ascension by 0$''$6 in declination. For the measurements presented here this active field of view is separated into, and nearly spanned by, 12 zones of equal width of declination, each covering the full reimaged aperture in the other dimension. The zones are defined by 13 parallel lengths of 12.7-mm wide steel shim stock spaced at 1.6-mm intervals, planes parallel to the optical axis, and one edge of each in the R' plane perpendicular to the declination dimension. The R' edge of the shims is sharpened on both sides at a 10$^\circ$ angle and polished, which reduces to a few arc seconds the insensitive region between zones. Twelve Plexiglas light pipes placed behind the opposite edges of the shims view the light selected for each zone and transmit it to the photomultipliers (PMs). Twelve PMs are arranged in two rows of six, one row of which is visible in Figure 2. Thus, each PM views a section of sky 0$''$38 wide in declination by 1$''$2 long in right ascension, and the array covers a total width of 0$''$45 in declination. The light pipes not only bring the light to the PMs but also spread and scramble it so the photocathodes are exposed in a roughly standard way as the stars move across the field of view.

Because the stars move along circles centered in the sky at the poles, the resulting bar of light at R' created by lens L3 moves within the area defined by the shims as a transit progresses, unless the declination is close to zero. With fully open light pipes, as at present, a star can stay within one PM response zone for declinations less then 85$''$; closer to the poles than this, all the stars move from PM to PM during their transits.

Availability of high-speed personal computers and interface boards is the major development that makes economical the type of telescope system presented here. The 12 Thorne-EMI 9635QA PM outputs are dc-coupled to the inputs of analog-to-digital converters, which sample and hold the integrated input each 75 milliseconds when strobed by a master oscillator. The digitization provides a 12-bit output word for each PM. A capacitor placed across each analog input determines an RC integration time, usually chosen to be close to the periodic sample time. The present 50-millisecond integration time is about 1/15 of the ruling-line-pair passage time at zero declination. Two I/O boards read out the analog-to-digital converters and the computer writes the results, with time information, to a 40-megabyte hard disc. This data-recording system recently replaced a previous microprocessor controller, which omitted data for three seconds every two minutes, while writing to video tape.

The Ronchi ruling has a fused silica substrate and eight line pairs per millimeter, so each line subtends 5.3 arc seconds in the sky. Examination of this ruling for irregularities with a measuring microscope shows no systematic error larger than 1 micron, but occasional holes in the opaque portions and dust specks in the transparent portions exist. These glitches occupy less than 0.001 of the open area of the ruling and are, in most cases, considerably smaller than the image size. They cause a negligible error contribution compared with the atmospheric seeing. This ruling has equal opaque and transparent lines, with widths chosen close to the maximum coma-dominated image length at both edges of the 1$''$ useful field of view for the telescope. The modulation fraction varies between 90% and 95% over the field of view. The "Ronchi frequency" is 1.4 Hz at zero declination.

Pupil A of the telescope has a diameter of 29.2 cm. It is not located at the primary mirror but in front of it by 3/4 of a focal length. This departure from usual practice reduces the maximum coma-dominated image length. As a price for this reduction, aperture A is 2.5 cm smaller than the M1 diameter, and thermal or temporal figure changes in
both M1 and M2 become a potential source of systematic error, and not just M2 as would have been the case had the entrance pupil been placed at M1.

6. Data Analysis and Results

Figure 3 shows typical samples of ruling-modulated starlight. The time-series cycles are similar throughout a transit, but asymmetry is present toward the beginning and end, due to the image coma. Because the telescope remains fixed and the ruling is not moved during the measurements, the rate at which the ruling lines are crossed is proportional to the secant of the declination of the object. The Ronchi frequency exhibits the expected declination dependence to within one part in 10,000. The proper confinement of the optical axis to the meridian plane has been confirmed, to an accuracy of about 1/2 arc minute, by visually comparing absolute transit times for stars at a range of declinations to the times predicted by the Astronomical Almanac.

The centroid of each Ronchi cycle is calculated, relative to a regularly spaced predicted transit time, for each transparent ruling line. The resulting time difference is used to define a value of “Ronchi phase” for that particular section of data, relative to zero phase at the beginning of the section of data being analyzed. In the absence of atmospheric seeing, a plot of Ronchi phase as a function of time is independent of time. Figure 4 shows the Ronchi phase, in arc seconds, for ζ Bootis and for a neighboring star separated in angle from it by 0°2. The variations in Figures 4(a) and 4(b) depart from a random distribution. Many of the same structures appear in both transits and each shows the characteristic “pink noise” spectrum described by Lindegren (1980). Figure 4(c) shows the result, when Figure 4(b) is subtracted from Figure 4(a) to yield the right-ascension projection of the differential angle between the two stars, modulo the angle subtended by a ruling cycle. Figure 4(c) shows that most of the pink character of the noise has been removed by the subtraction.

The data sequence of Figure 4(c) has been averaged in various integration time intervals and the resulting distributions used to infer the dependence of precision on integration time. The resulting precision is presented in Figure 5, both for absolute and differential star coordinates. A comparison was also made between five nights of data recorded from 1989 June 9 to 13. The pair 48 Cygni was chosen for brightness (6.2 B8 and 6.5 F0) and proximity (adjacent light pipes, declination separation = 180 arc seconds, right-ascension separation = 13 arc seconds). All data presented here have stars sufficiently bright that photodetector counting statistics play no role in limiting the performance. Figure 5 shows the results of this comparison for integrations of varying lengths, up to 282 seconds of the 325-second transits, while the stars were simultaneously in the desired detectors. The resulting single-night precision of 3.6 milli-arc-seconds, for about 300 seconds of integration, meets the expected single-star 4-milli-arc-seconds design specification for the apparatus.

7. Discussion

The realization of full differential precision for an optimum bright pair of stars meets the first objective in
establishing the suitability of this instrument for an astrometric observation program. As a next step, sequences of data as illustrated in Figure 4, with a range of angles separating the pairs of stars, can explore the isoplanatic patch, out to about 0.5° in angle. Analysis of the isoplanatic patch data is in progress and will be presented elsewhere. We note preliminary results of this kind with the Allegheny Observatory instrument (Han 1989).

The faintest limiting magnitude for this telescope is determined by the aperture area, the optical transmission efficiency, the photoelectric conversion efficiency, the integration time, and the amount of largely unmodulated background light and dark current for each PM. In the absence of this background, the system would realize a single-transit precision of 4 milli-arc-seconds when the error contribution from photoelectron statistical fluctuations within a complete transit is smaller than about 0.001 of a ruling linewidth. For the full precision to be realized this means, roughly, that more than 150,000 photoelectrons must be detected during a transit. Using the present values of the system (optical/ruling transmission efficiency of 10%, photoelectric conversion efficiency of 20%, and 300 seconds of integration time) yields about 12th magnitude as the brightness at which photoelectron fluctuations from the star alone begin to compete in the present system with atmospheric seeing.

Background light is present, however, due to the presence of many faint stars and, sometimes, light scattered from the Moon or from cities. This light is typically less than 10% modulated at the Ronchi frequency because the
faint stars have differing Ronchi phases. Fluctuations in this background contribute to astrometric error. To date the telescope has been operated only with the light pipes fully open, and the resulting typical sky radiation is equivalent to a 7th- to 8th-magnitude star. The distribution of apparent object positions, as indicated by individual Ronchi cycles, begins to broaden for stars fainter than this; 4-milli-arc-second differential precision is expected to persist for stars brighter than magnitude 8 to 8.5. A related effect occurs due to the portion of background light modulated at the Ronchi frequency. This simply offsets the measured angles of the target and reference stars throughout a series of measurements. Such an offset remains constant through a series of differential measurements, if the telescope is placed back at the same location in the sky each time.

The performance of this instrument can be improved by several modifications. A modification of this telescope could remove cylindrical lens L3 so both coordinates are reimaged at R' and couple the photomultipliers to small fiber light pipes which would then be moved along the right-ascension coordinate in the R' plane at the proper speed to follow the stars. At the cost of introducing moving parts during the transits, this alteration would render negligible the background light in each PM. Another potential light-pipe modification involves introduction of
the light with multiple internal reflections in the PM face (Buffington et al. 1977), yielding a significant enhancement of photocathode conversion efficiency. Adoption of all-reflecting relay optics (Offner 1975) could further enhance efficiency, possibly yielding a total factor of five for these latter two improvements.

A significant advantage of using large area detectors is that the sky is almost entirely covered in the declination dimension. Thus, when a rapidly moving solar-system object is the target, the telescope can be locked down and not disturbed for several to many days, as required, nor is it necessary beforehand to know the precise position of the target. An additional advantage is that many reference stars occur within a single light pipe, and not a single one as would have been the case with a light pipe viewing a small area of sky around each chosen star. This increases the number of photoelectrons contributing to establishing a reference frame and opens the possibility of tracking the seeing for a substantial period encompassing the transit time of the target object, without compromising the symmetrical integration for the target around the meridian plane. The cost of such fully open light pipes is a substantial amount of background light whose fluctuations limit the faintest object, target or reference, for which the instrument can deliver full precision. Another disadvantage is the need to return to a chosen sky field to a precision that depends on the magnitude of the desired star but is typically better than an arc second.

This work was supported in part by a NASA grant while one of us (A.B.) was at the California Institute of Technology, in part by grants from the Lunar and Planetary Laboratory at the University of Arizona, and from the California Space Institute, and in part by subcontracts from the Jet Propulsion Laboratory. Present space is too limited to acknowledge the many people who have helped with contributions of equipment, ideas, and volunteer labor. However, particular thanks should be given to Bill Althouse, Hans Grau, John Jacobs, Sven Sondgaard, and Joe Ungerer for help in building the apparatus; to Steen Laursen of the Danish Space Research Institute for electronics help; to Dave Carr, Bob Howard, Tony Misch, and Larry Webster for help at Mount Wilson; to Jerry Cook, Robert Geller, Corie Jacobs, John Knox-Seith, Elena Mendoza, and Marc Raymon, who helped erect the buildings and get the telescope operational in its location at Table Mountain; and to Corwin Booth for interfacing the personal computer. We have benefited from numerous conversations with George Gatewood, Eugene Levy, Bob McMillan, Marc Raymon, Bob Reasenberg, Richard Rouse, Bonny Schumaker, and John Stein. We further thank Bob Reasenberg for a thoughtful critical reading of this manuscript. At U.C.S.D., we are grateful for mainframe computer access courtesy of Carl Mcllwain and for additional computer support provided by the group of Jon Matteson and Larry Peterson. We acknowledge the hospitality of the Carnegie Institution while the apparatus was operational at Mount Wilson and of the Jet Propulsion Laboratory for its support of our operations at Table Mountain.

APPENDIX

Proof that Image Centroid Coincides with Response Time Series Mean

In this Appendix we prove that the centroid and the mean of a response time series \( R(t) \) coincide, for an \( R(t) \) generated by transmission of an image through a slot whose width is greater than the size of the image. Strictly speaking, when diffraction is included, the image "size" cannot be thus bounded, and as a rough estimate the equivalence proven here is limited by the extent that the full image is not transmitted at some one position of the slot.

Let the image intensity be \( i(x,y) \) in a plane containing a scanning slot of width \( x_o \). At time \( t = 0 \), let the slot be placed with its center at \( x = 0 \), and its edges aligned parallel with the \( y \) axis, and its motion along \( \pm x \), so the \( y \) dimension can be suppressed:

\[
 i(x) = \int_{-\infty}^{\infty} i(x,y) \, dy . \tag{A1}
\]

Consider further that the slot moves toward \( +x \) with increasing time, so for \( t < 0 \) the right-hand edge of the slot is cutting off the portion of the image with \( x \) greater than \( x_o/2 + vt \), while for \( t > 0 \), the left-hand edge of the slot is cutting off the portion of the image with \( x \) less than \( -x_o/2 + vt \). The \( x \) coordinate can also be represented in terms of the time at which the center of the slot passes a particular location, \( x = vt \), so an \( i(t) \) is easily scaled from equation (A1). In this coordinate, the image centroid is given by

\[
 \langle T \rangle = \int_{-\infty}^{\infty} \frac{t \, i(t) \, dt}{\int_{-\infty}^{\infty} i(t) \, dt} ,
\]

\[
 = \int_{-\infty}^{\infty} \frac{t \, i(t) \, dt}{\int_{-\infty}^{\infty} i(t) \, dt} , \tag{A2}
\]
where the limits in the second line include the assumed finite size of the image, and \( T = \frac{x}{v} \) is the time for an edge to cross the full width of the image.

A response time series \( R(t) \) is generated by the integrated light from the image passing through the slot. The full nonzero domain for \( R(t) \) is \(-T < t < T\), twice the domain for the image itself. The image intensity \( i(t) \) can be represented as a sum of powers of \( t \), with the first coefficient \( a_0 \) the intensity at \( t = 0 \):

\[
i(t) = \sum_{j=0}^{\infty} a_j t^j.
\]

(A3)

In this representation, equation (A2) can be rewritten

\[
\langle T \rangle = \int_{-T/2}^{T/2} \sum_{j=0}^{\infty} a_j t^{j+1} \, dt / \int_{-T/2}^{T/2} \sum_{j=0}^{\infty} a_j t^j \, dt,
\]

\[
= \sum_{j=0}^{\infty} \frac{2a_j}{j+2} \frac{(T/2)^{j+2}}{(T/2)^j} = \sum_{j=0}^{\infty} \frac{2a_j}{j+1} \frac{(T/2)^{j+2}}{(T/2)^j},
\]

(A4)

where the odd powers of \( t \) cancel in the integrals because of the symmetrical limits, and the summation symbols are limited to odd or even values of \( j \):

\[
\sum^o \text{ implies only odd values for } j; \sum^* \text{ only even values.}
\]

The response time series can now be derived in terms of integrals over \( i(t) \) and its power series expansion given by equation (A3). At \( t = 0 \), \( R(t) \) assumes its greatest value \( R_\infty \) since the slot at this time transmits all the light of the image.

\[
R_\infty = R(0) - \int_{-T/2}^{T/2} i(t) \, dt = \sum_{j=0}^{\infty} \frac{2a_j}{j+1} \frac{(T/2)^{j+1}}{(T/2)^j}.
\]

(A5)

For \( t = 0 \) a portion of the light is obscured by either the right- or left-hand edge of the slot, and

\[
R(t) = R_0 \begin{cases} \int_{-T/2-t}^{T/2-t} i(t') \, dt' & -T < t < 0 \\ \int_{-T/2}^{T/2} i(t') \, dt' & 0 < t < T \\ 0 & T < t \end{cases}
\]

\[
= R_0 \sum_{j=0}^{\infty} \frac{a_j}{j+1} \left( (t-T/2)^{j+1} - (T/2)^{j+1} \right) & -T < t < 0 \\
= R_0 \sum_{j=0}^{\infty} \frac{a_j}{j+1} \left( (t-T/2)^{j+1} - (T/2)^{j+1} \right) & 0 < t < T.
\]

(A6)

The mean of \( R(t) \), the "centroid of the response time series" is given by:

\[
\langle R \rangle = \int_{-T/2}^{T/2} t \, R(t) \, dt / \int_{-T/2}^{T/2} R(t) \, dt
\]

\[
= \frac{1}{2 R_\infty T} \sum_{j=0}^{\infty} \frac{a_j}{j+1} \left( \int_{-T/2}^{T/2} (t-T/2)^{j+1} \, dt - \int_{-T/2}^{T/2} (T/2)^{j+1} \, dt \right)
\]

\[
= \frac{1}{2 R_\infty T} \sum_{j=0}^{\infty} \frac{a_j}{j+1} \left( \int_{-T/2}^{T/2} (t-T/2)^{j+1} \, dt - \int_{T/2}^{T/2} (T/2)^{j+1} \, dt \right)
\]
where several terms have canceled in both numerator and denominator, finally yielding for each the factor $T$ times the numerator and denominator in equation (A4), so the equivalence of the two quantities is proven.

REFERENCES

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