ASTR1010 - INTRODUCTION TO ASTRONOMY

The Solar System

February 23, 2009

EINSTEIN'S THEORY OF RELATIVITY – II

Introduction

In your high school and/or college math classes you have seen the x-y plane many times. The coordinates associated with this plane are called Cartesian coordinates after Rene Descartes, a contemporary of Galileo who explored the properties of this two-dimensional "space". You also have seen how you can add a third axis perpendicular to the other two (i.e., coming out and going into the plane of the page) to create the x-y-z plane which is the Cartesian coordinate system in a three-dimensional Euclidean space. What is this coordinate system useful for? Well, it defines a reference system that can be used to identify the components of objects called vectors. You might remember that a vector is a mathematical object that has magnitude and direction (it's often depicted as an "arrow"). If you have a vector in your 3-d coordinate space, then you can break down its length along the x-axis, the y-axis, and the z-axis. Those lengths are called the components of the vector and when they are added together in the following fashion, they give the overall total length of the vector:

$$\Delta x^2 + \Delta y^2 + \Delta z^2 = (\text{the length})^2$$

In this formula (just an extension of the famous Pythagorean Theorem), the symbols Δx Δy, and Δz represent the distance or displacement along the x, y, or z axis. We will call the total length of the vector "s" so that $\Delta x^2 + \Delta y^2 + \Delta z^2 = s^2$. In case you're wondering, we could figure out the direction of the vector if we knew the values of Δx, Δy, and Δz, but we don't care about that for this lesson. Anyway, we now have a prescription for figuring out the length of any vector in any 3-d Cartesian coordinate system, but why is this useful? Well, it is important to recognize that it is possible to rotate the coordinate system and the length of the vector will not change. The components of the vector change, but the length stays the same no matter how we shift or rotate the coordinate system. The 2-d figure below illustrates this principle: