Application Exercise: Distances to Stars Using Measured Parallax

Objectives

- Qualitatively state relative distances to stars based upon their parallax shift
- Describe the limits of the measured parallax method of determining distances to stars
- Find the mathematical relationship between the measured parallax of a star and its distance in parsecs
- Given the values of measured parallaxes for a list of stars, calculate the distances
- Measure the parallaxes of two stars, calculate the distances, and state the uncertainties in the measurements
- Summarize the method, giving an evaluation of its strengths and weaknesses

Materials

Scientific calculator  
Rulers

Introduction

One of the most difficult problems in astronomy is determining the distances to objects in the sky. There are four basic methods of determining distances: radar, parallax, standard candles, and the Hubble Law. Each of these methods is most useful at certain distances, with radar being useful nearby (e.g., the Moon), the Hubble Law being useful at the farthest distance (e.g., galaxies far, far away). In this exercise, we investigate the use of the measured parallax method to determine distances to nearby stars, those within about 650 light years from the Sun.

Even when observed with the largest telescopes, stars are still just points of light. Although we may be able to tell a lot about a star through its light, these observations do not give us a reference scale to use to measure its distance. We need to rely on a method that you are familiar with: the parallax of an object.

You can see the parallax effect by holding a finger out at arm’s length. View your index finger relative to a distance background while you alternate opening and closing each eye. Does your finger seem to jump back and forth relative to the background? This is because the centers of your eyes are 5 – 6 centimeters apart, so each eye has a different point of view.

Because stars are so far away, and their parallax shifts so extremely small, the parallaxes are most conveniently measured in seconds of arc (arc seconds). The angular size of your index finger held at arm’s length is about 1 degree. Imagine dividing this finger up into 3600 slices. One of these slices would represent the angular size of an arc second!

Without today’s advanced observing techniques, measuring the parallax angle of even the closest star is impossible. The teachings of Aristotle (384 – 322 BC) and the mathematical model of Ptolemy (c. 140 AD) based upon Aristotle’s universe, formed the foundation of astronomy for almost 1500 years. This universe was geocentric; it placed the Earth at the center (corrupt and changeable) of the heavens (perfect and immutable). Those astronomers who suggested that the Earth orbited the Sun were dismissed with the argument that if the Earth did orbit the Sun, the stars would show annual shifts, or a parallax. But, “…no matter how hard they searched, ancient astronomers could find no sign of stellar parallax.” (Bennett et al. The Cosmic Perspective, Addison Wesley, 2002)
Part I: Sample Observations

01. <2 pts> Represented here are observations of two different stars, Abba and Babba. Each is observed in January and then again in July; the observations are 6 months apart. Which star is closer to us? Explain how you know that based simply off of the picture.

02. <2 pts> Let’s say that star Abba has a parallax angle about two times that of star Babba. What can we immediately determine about the relative distances of the two stars? Explain how we know this.

03. <2 pts> What if there were a star that had a parallax angle 1000 times smaller than that of star Babba? Assuming no advancement in our technology, would we still be able to detect a difference in its shift between January and July? What does this tell us about the limits of measuring parallaxes?
04. <2 pts> The limit to our measurements of parallax angles is about 0.005 arc seconds, or about 650 light years. The diagram at the right represents the Milky Way from a top-down view. Draw a circle on this diagram around the location of the Sun that indicates the range of usefulness of measured parallax. To what fractional part of the diameter of the Milky Way can we determine distances using this method?

Part II: Mathematical Explanation of Parallax

05. <2 pts> One parsec is defined to be the distance to a star whose parallax angle is one arc second. If we have a star whose parallax angle is ½ of an arc second, its distance is 2 parsecs. If the star has a parallax angle of 3 arc seconds, its distance is 1/3 of a parsec. Based on this and the tutorial for this exercise, what is the mathematical relationship? (A word description is fine as well.)

06. <2 pts> The graph to the left shows the relationship between measured parallax and distance. Does the relationship match the mathematical description you found above? Check your answer from question 5 with this graph by choosing a parallax value and calculating the corresponding distance.
07. <2 pts> Below is a list of stars and their parallaxes. Rank use 1-4 where 1 is the nearest and 4 is the farthest.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Star Name</th>
<th>Parallax in arc seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Antares</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td>Ross 780</td>
<td>0.213</td>
</tr>
<tr>
<td></td>
<td>Regulus</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>Betelgeuse</td>
<td>0.009</td>
</tr>
</tbody>
</table>

08. <3 pts> Complete the table below by calculating the distances to the stars. The formula for calculating the parallax is simply: \( d = \frac{1}{p} \), where \( d \) is the distance in parsecs, and \( p \) is the measured parallax in arc seconds. Expressed another way: the distance in parsecs is just the inverse of the parallax in arc seconds.

<table>
<thead>
<tr>
<th>Star Name</th>
<th>Parallax (arc seconds)</th>
<th>Distance (parsecs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arcturus</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>Procyon</td>
<td>0.288</td>
<td></td>
</tr>
<tr>
<td>Hadar</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>Rigel</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>Sirius</td>
<td>0.379</td>
<td></td>
</tr>
<tr>
<td>Altair</td>
<td>0.194</td>
<td></td>
</tr>
</tbody>
</table>

09. <3 pts> The image below is a sample observation of two stars in the same field observed 6 months apart. We combined two images so you could see the shift the stars made in this time. The scale in the upper right corner represents an angle of 0.1 arc seconds (not 0.1 inches).

What is the total angular shift for each star over this time period?
A _______ B_______

What is the parallax angle for each star?
A _______ B_______

What is the distance to each star, in parsecs?
A _______ B_______

III. Scientific Uncertainties in measurements

10. <2 pts> How uncertain are you in your results for question 9 using this method? That is, how far off could your values be due to measurement errors? Discuss briefly why there is this uncertainty.
<table>
<thead>
<tr>
<th>Star Name</th>
<th>Parallax Angle</th>
<th>Distance</th>
<th>Distance Range</th>
<th>Uncertainty Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(arcsecs)</td>
<td>(parsecs)</td>
<td>(light-years)</td>
<td></td>
</tr>
<tr>
<td>Ain (ε Tauri)</td>
<td>0.021</td>
<td>48</td>
<td>155</td>
<td>149-161</td>
</tr>
<tr>
<td>Bellatrix (γ Orionis)</td>
<td>0.013</td>
<td>75</td>
<td>243</td>
<td>226-262</td>
</tr>
<tr>
<td>Spica (α Virginis)</td>
<td>0.012</td>
<td>80</td>
<td>262</td>
<td>245-282</td>
</tr>
<tr>
<td>Betelgeuse (α Orionis)</td>
<td>0.009</td>
<td>113</td>
<td>368</td>
<td>352-544</td>
</tr>
<tr>
<td>Polaris (α Ursae Minoris)</td>
<td>0.008</td>
<td>132</td>
<td>431</td>
<td>405-460</td>
</tr>
<tr>
<td>Antares (α Scorpii)</td>
<td>0.007</td>
<td>144</td>
<td>469</td>
<td>460-876</td>
</tr>
<tr>
<td>Rigel (β Orionis)</td>
<td>0.004</td>
<td>237</td>
<td>773</td>
<td>648-956</td>
</tr>
<tr>
<td>Deneb (α Cygni)</td>
<td>0.002</td>
<td>660</td>
<td>2,150</td>
<td>2,063-7,409</td>
</tr>
</tbody>
</table>

11. <3 pts> Review the information in the above table. What do you notice about the relationship between the distance to a star and the uncertainty connected with that distance measurement? Discuss briefly whether or not this trend makes sense, and why astronomers would still use a measured parallax distance even though the uncertainty is very high.

Part IV: Summary

12. <5 pts> Explain to an incoming Astronomy 101 student about measured parallax: What are measured parallaxes used for? For how far away are they useful? How does one measure a stellar parallax? What are the strengths and weaknesses of this method? Use good writing style in this summary.