Frustration, dimension and spin anisotropy: The route to quantum disorder on the Sierpinski Gasket antiferromagnet

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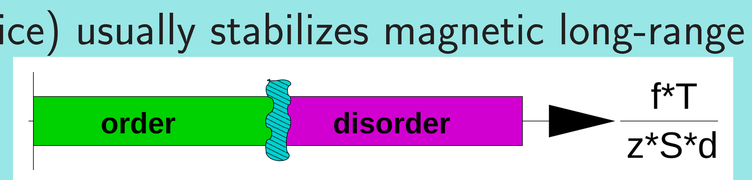
1 Introduction

We study the ground state magnetic order of low-dimensional quantum antiferromagnets (AFM) with a variety of parameters. The increase of parameters like temperature or frustration may drive the system into a disordered state. The increase of spin quantum number, dimension or coordination number (number of neighbor spins on a lattice) usually stabilizes magnetic long-range order (LRO).



The transition between one- and two-dimensional systems has been studied in particular because of the fundamental difference of the magentic order in the ground state. The d=1 linear chain show no LRO whereas several d=2 lattices like square, triangular or honeycomb lattices show a Néel-like magnetic ground state.
We have studied the quantum Heisenberg AFM on a Sierpinski gasket with a fractal dimension between one and two. For a Heisenberg interaction between the nearest-neighbor spins on this lattice we have presented arguments in favor of a disordered ground state [1, 2].

2 Model

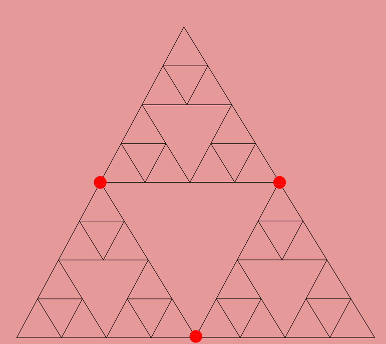
We consider the quantum $s = \frac{1}{2}$ AFM with anisotropic spin exchange.

$$\hat{H} = J \sum_{i,j} (\mathbf{S}_i^x \mathbf{S}_j^x + \mathbf{S}_i^y \mathbf{S}_j^y) + \Delta \mathbf{S}_i^z \mathbf{S}_j^z$$

The spin exchange is antiferromagnetic $J > 0$, the anisotropy is given by Δ with $\Delta = 1$ for the Heisenberg model and $\Delta = 0$ for the XY model. The main geometrical property of the lattice is its fractal Hausdorff dimension of $d_f = \frac{\ln(3)}{\ln(2)} \approx 1.58$. The number of spins on this lattice is given by $N = \frac{1}{2}(3^n + 3)$ where $n = 1, 2, 3, 4$ corresponds to $N = 3, 6, 15, 42$.

3 Methods

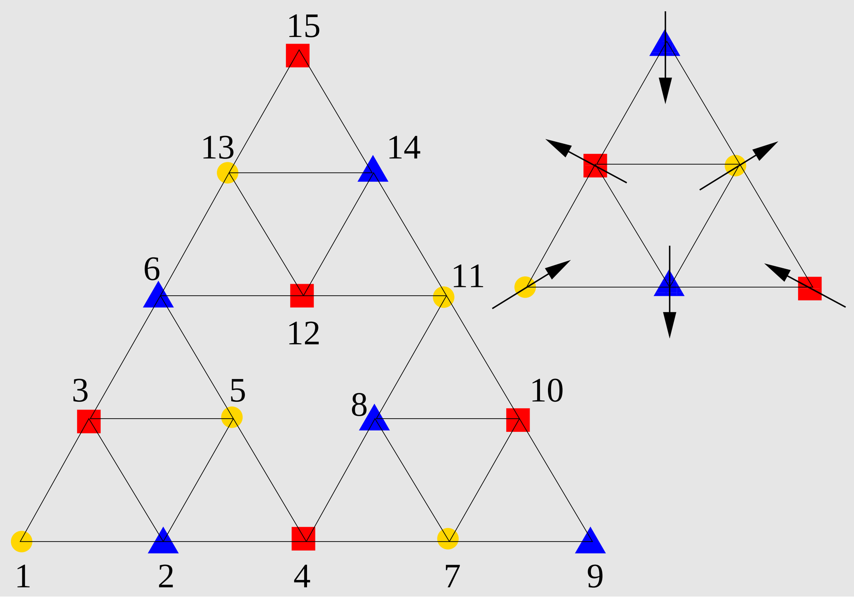
Several numerical methods have been used so far to calculate properties of the model: ground state and low energy excitations with Lanczos diagonalization, low temperature thermodynamics with full diagonalization and a quantum decimation technique. Additionally a new method, the configuration selective diagonalization (CSD), has been developed by W. Wenzel.
Using ideas of quantum chemistry [3] the CSD is based on a iterative procedure where the wave function will be a product state of uncoupled fragments of the lattice (in the figure below the sites with a red dot are used to decouple the lattice into three identical fragments). These fragments are pertubatively coupled and only the most important configurations are kept for an iterative procedure [4]. The method thus consists of two basic steps, the *expansion* (where configurations are selected) and the *diagonalization* (where the corresponding eigenvalues are calculated).



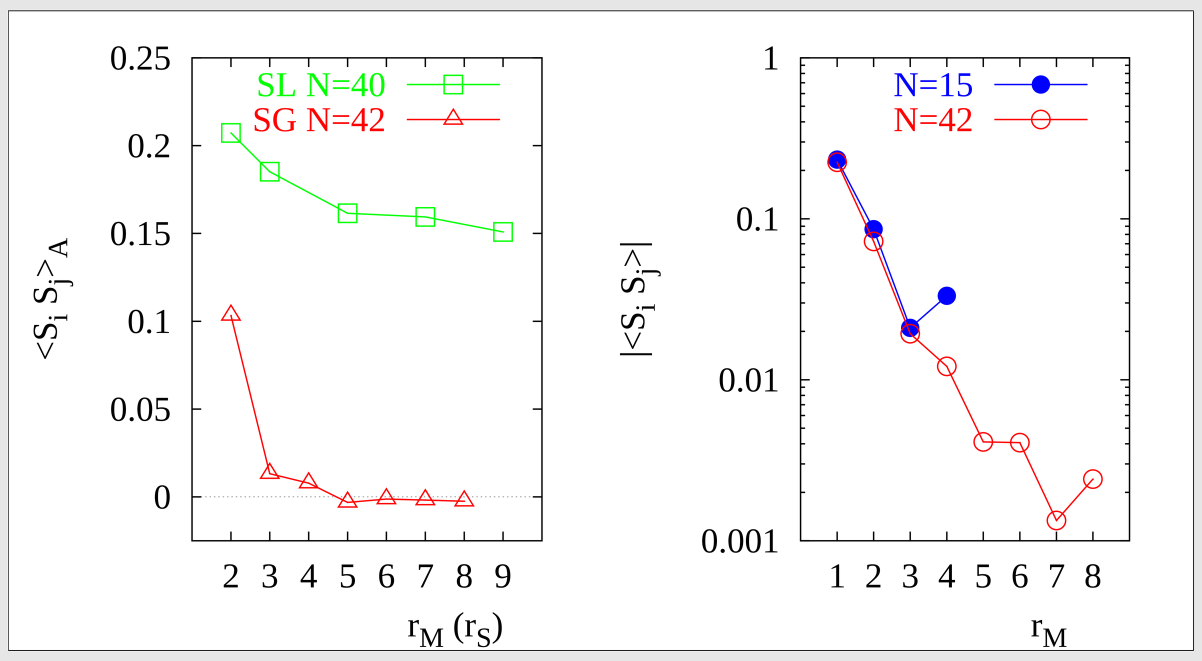
4 Results

4.1 Classical ground state

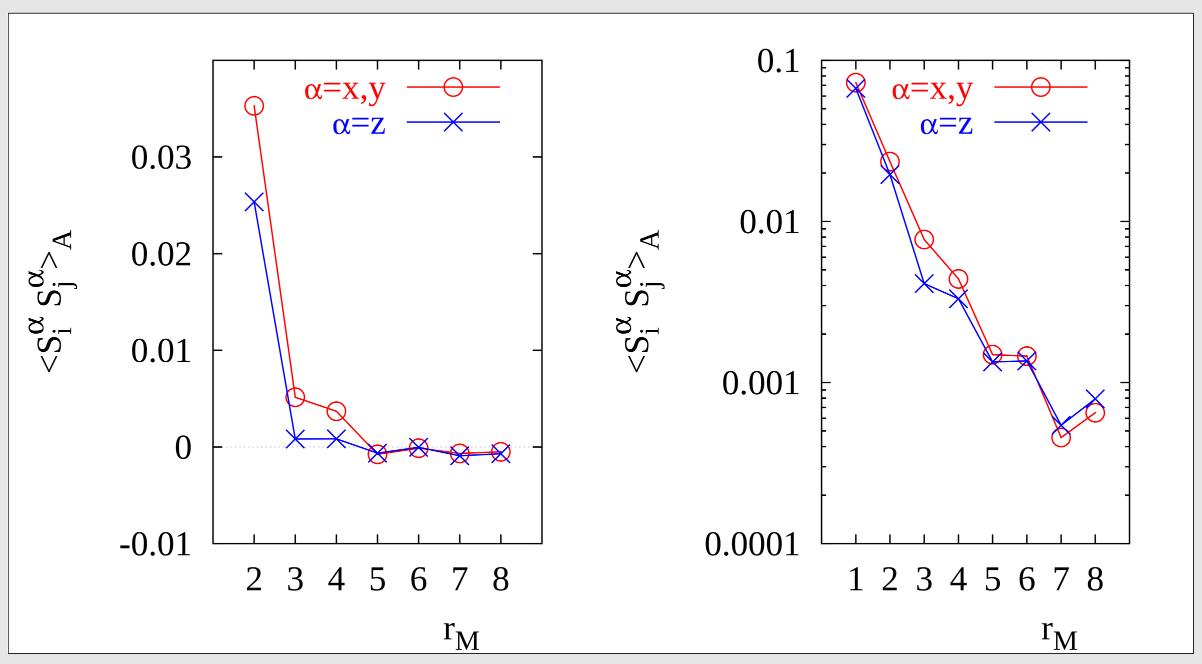
The classical ground state configuration of the Sierpinski gasket has three sublattices. The spins of a subsystem are ferromagnetically aligned, between different sublattice spins one observes a 120° structure.



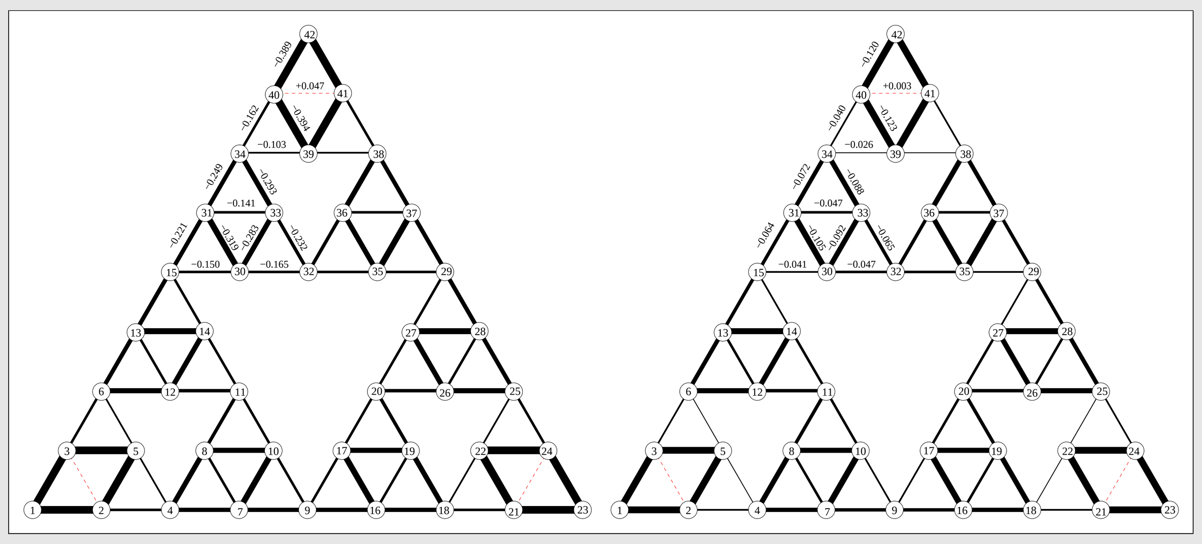
4.2 Spin-spin correlations $\langle \mathbf{S}_i^\alpha \mathbf{S}_j^\alpha \rangle$ $\alpha = x, y, z$



$\Delta = 1$: For the Heisenberg case the sublattice spin-spin correlations over the Manhattan distance (left) show that the classical order is completely lost (see in contrast the almost constant correlation behavior for a square lattice). The absolute value of all spin-spin correlations (right) are exponentially decreasing supporting the previous conclusions about a disordered ground state.

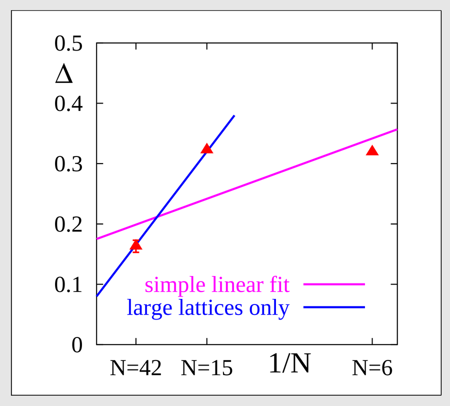


$\Delta = 0$: For the XY case the different components of the spin-spin correlations behave quite similar, the classical order has vanished and the absolute spin-spin correlations are pointing to a disordered ground state as well.



The nearest neighbor spin-spin correlations show the building of strong local plaquettes for $\Delta = 1$ (left) and $\Delta = 0$ (right). A similar behavior has been observed already for N=15 and for other strong frustrated models (like in the $J_1 - J_2$ square lattice model).

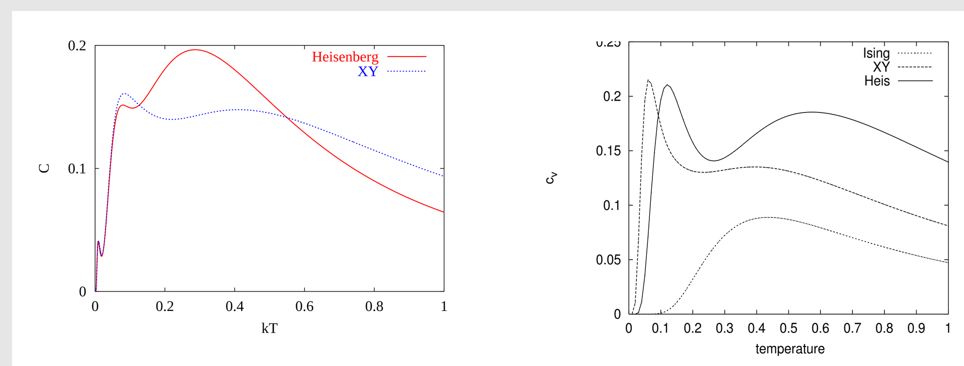
4.3 Spin gap $\Delta = E(S_{min} + 1) - E(S_{min})$



The CSD method allows the calculation of excited states as well, therefore the spin gap has been calculated. We argue that the spin gap remains finite which is another indication for a disordered ground state.

4.4 Thermodynamics

The specific heat for the N=15 lattice shows an additional low-temperature peak (left) and corresponding calculations with a quantum decimation technique confirm this observation (right). This behavior is consistent with a spin gap supporting the previous observations of a disordered ground state.



5 Conclusion

Using several numerical methods and a newly developed configuration selective diagonalization approach we investigate the ground state magnetic order on the Sierpinski gasket. The interplay of frustration and fractal dimension leads to a disordered ground state. This behavior is independent of a XY spin anisotropy.

References

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