Kinematics in two dimensions
If \( \vec{a} \) is constant, then the x- and y-components of motion are independent of each other.

\[
x_f = x_i + v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2
\]
\[
y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2
\]
\[
v_{fx} = v_{ix} + a_x \Delta t
\]
\[
v_{fy} = v_{iy} + a_y \Delta t
\]

Newton’s Third Law
Every force occurs as one member of an **action/reaction pair** of forces. The two members of an action/reaction pair:
- Act on two **different** objects.
- Are equal in magnitude but opposite in direction:
  \[
  \vec{F}_{A\text{ on B}} = -\vec{F}_{B\text{ on A}}
  \]

**rtz-coordinates**
- The \( r \)-axis points toward the center of the circle.
- The \( t \)-axis is tangent, pointing counterclockwise.

Newton’s Second Law
Expressed in x- and y-component form:
\[
(F_{\text{net}})_x = \sum F_x = ma_x
\]
\[
(F_{\text{net}})_y = \sum F_y = ma_y
\]

Circular motion kinematics
\[
\text{Period } \quad T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}
\]
\[
\text{Angular position } \quad \theta = \frac{s}{r}
\]
\[
\omega_f = \omega_i + \alpha \Delta t
\]
\[
\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2
\]
\[
\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta
\]

Nonuniform Circular Motion
**Angular acceleration** \( \alpha = \frac{d\omega}{dt} \).
The radial acceleration
\[
a_r = \frac{v^2}{r} = \omega^2 r
\]
changes the particle’s direction. The tangential component
\[
a_t = \omega r
\]
changes the particle’s speed.

Angular velocity
\[
\omega = \frac{d\theta}{dt}
\]
\[
v_r = \omega r
\]

Angular acceleration
\[
\alpha = \frac{d\omega}{dt}
\]
\[
a_r = \omega r
\]

Expressed in \( rtz \)-component form:
\[
(F_{\text{net}})_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r
\]
\[
(F_{\text{net}})_t = \sum F_t = \begin{cases} 
0 & \text{uniform circular motion} \\
ma_r & \text{nonuniform circular motion}
\end{cases}
\]
\[
(F_{\text{net}})_z = \sum F_z = 0
\]
**Law of Conservation of Momentum**
The total momentum \( \vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots \) of an isolated system is a constant. Thus
\[
\vec{p}_t = \vec{p}_i
\]

**Newton’s Second Law**
In terms of momentum, Newton’s second law is
\[
\vec{F} = \frac{d\vec{p}}{dt}
\]

**Momentum**
\[
\vec{p} = m\vec{v}
\]

**Impulse**
\[
J_i = \int_{t_i}^{t_f} F_i(t) \, dt = \text{area under force curve}
\]

**Impulse and Momentum**
are related by the **impulse-momentum theorem**
\[
\Delta p_x = J_x
\]
The impulse delivered to a particle causes the particle’s momentum to change. This is an alternative statement of Newton’s second law.

**Basic Energy Model**
- Energy is **transferred** to or from the system by work.
- Energy is **transformed** within the system.

Two versions of the energy equation are
\[
\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{int} = W_{ext}
\]

\[K_i + U_i + E_{int} = K_f + U_f + W_{ext} \]

The work done by a force on a particle as it moves from \( s_i \) to \( s_f \) is
\[
W = \int_{s_i}^{s_f} F_i \, ds = \text{area under the force curve}
\]

= \( \vec{F} \cdot \Delta \vec{s} \) if \( \vec{F} \) is a constant force

**Conservative forces** are forces for which the work is independent of the path followed. The work done by a conservative force can be represented as a **potential energy**:
\[
\Delta U = U_f - U_i = -W_i(i \rightarrow f)
\]

A conservative force is found from the potential energy by
\[
F_s = -\frac{dU}{ds} = \text{negative of the slope of the PE curve}
\]

**Dissipative forces** transform **macroscopic energy** into thermal energy, which is the **microscopic energy** of the atoms and molecules. For friction:
\[
\Delta E_{th} = f_s \Delta s
\]