# The University of Georgia Department of Physics and Astronomy 

Prelim Exam<br>January 8, 2021

Part I (problems 1, 2, 3, and 4) 9:00 am - 1:00 pm

## Instructions:

- Start each problem on a new sheet of paper. Write the problem number on the top left of each page and your pre-arranged prelim ID number (but not your name) on the top right of each page. Leave margins for stapling and photocopying.
- Put your answers only on one side of the paper. Please do not write on the back side.
- If not advised otherwise, derive the mathematical solution for a problem from basic principles or general laws (Newton's laws, Maxwell's equations, Schrodinger's equation, etc.).
- You may use a calculator for basic operations only (i.e., not for referring to notes stored in memory, symbolic algebra, symbolic and numerical integration, etc.) The use of cell phones, tablets, and laptops is not permitted.


## Problem 1

The figure below shows a 200 g uniform rod pivoted at one end. The other end is attached to a horizontal spring. The spring is neither stretched nor compressed when the rod hangs straight down.


What is the rod's oscillation period?
(You can assume that the rod's angle from the vertical is always small.)

## Problem 2

Consider a rocky planet with uniform mass density $\rho$. If the planet rotates too fast, it will fly apart.

a) By considering centrifugal force and gravity at the equator, show that the minimum period of rotation is $T=\sqrt{3 \pi / G \rho}$.
b) Show that the limiting period is the same $(T=\sqrt{3 \pi / G \rho})$ at a latitude of $\varphi$.
c) What is the minimum $T$ for the Earth? (Earth's average density is $5.5 \mathrm{~g} / \mathrm{cm}^{3}$, $G=6.67 \times 10^{-11} \mathrm{~m}^{3} / \mathrm{kg} \mathrm{sec}{ }^{2}$ )

## Problem 3

A long solenoid of radius $a$, carrying $N$ turns per unit length, is looped by a wire of resistance $R$ (see Figure below).

a) If the current in the solenoid is increasing, $\mathrm{dI} / \mathrm{dt}=\mathrm{k}=$ constant, what current flows in the loop, and which way ( $a$-to- $b$ or $b$-to- $a$ ) does it pass through the resistor?
b) If the current $I$ in the solenoid is constant, but the solenoid is pulled out of the loop and reinserted in the opposite direction, what total charge passes through the resistor?

## Problem 4

Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a tank of dielectric oil (susceptibility $\chi_{\mathrm{e}}$, mass density $\rho$ ). The inner one is maintained at potential V , and the outer one is grounded.


To what height (h) does the oil rise in the space between the tubes?

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## Part II (problems 5 and 6) 2:00 pm - 4:00 pm

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- If not advised otherwise, derive the mathematical solution for a problem from basic principles or general laws (Newton's laws, Maxwell's equations, Schrodinger's equation, etc.).
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## Problem 5

A particle confined in a rigid one-dimensional box of length 10 fm has an energy level $\mathrm{E}_{\mathrm{n}}=32.9 \mathrm{MeV}$ and an adjacent energy level $\mathrm{E}_{\mathrm{n}+1}=51.4 \mathrm{MeV}$.
a. Determine the values of n and $\mathrm{n}+1$.
b. Draw an energy-level diagram showing all energy levels from 1 through $\mathrm{n}+1$. Label each level and write the energy beside it.
c. Sketch the $\mathrm{n}+1$ wave function on the $\mathrm{n}+1$ energy level.
d. What is the wavelength of a photon emitted in the $n+1 \rightarrow n$ transition? Compare this to a typical visible-light wavelength.
e. What is the mass of the particle? Can you identify it?

## Problem 6

A particle of mass $m$ is confined to a ring of radius $r_{0}$ centered on the origin, on which it moves freely (i.e., $V(\phi)=0$ ). The Hamiltonian is:

$$
\mathcal{H}_{0}=-\frac{\hbar^{2}}{2 m r_{0}^{2}} \frac{d^{2}}{d \phi^{2}}
$$

(a) Solve for the energy eigenvalues and for the normalized energy eigenfunctions as a function of the azimuthal angle, $\phi$. Show that the states are discrete (i.e., the energy eigenstates can be indexed by an integer, $n$ ), that the ground state is nondegenerate, and that all other energy levels are twofold degenerate. Physically, what accounts for this degeneracy?
(b) A small perturbation of the form:

$$
\mathcal{H}^{\prime}=\epsilon \phi(\phi-\pi)(\phi-2 \pi)
$$

is applied, where $\epsilon$ is a small energy. Calculate the lowest-order nonzero correction to each energy level in perturbation theory. For degenerate unperturbed states, find the linear combinations that are eigenstates of the perturbation as well.

