

## January 2006 Written Comprehensive Exam - Day 1

This is a closed-book, closed-note exam. You may use a calculator, but only for arithmetic functions (not for referring to notes stored in memory). Attempt all six problems. Start each problem on a new sheet of paper and use one side only. Print your name on each piece of paper that you submit. For full credit you must show your work and/or explain your answers.

**Problem 1** (one part)

A rope of mass total  $M$  and length  $L$  is suspended vertically with its lower end touching a scale. The rope is released and falls onto the scale. What is the reading on the scale when a length  $x$  of the rope has fallen?

**Problem 2** (three parts)

An empty freight car of mass 10000 kg rolls at 2 m/s along a level track and collides with a loaded car of mass 20000 kg, standing at rest with brakes released.

(a) If the cars couple together, what is their speed after the collision?

(b) Find the decrease in kinetic energy as a result of the collision.

(c) With what speed should the loaded car be rolling toward the empty car for both to be brought to rest by the collision?

**Problem 3** (one part)

Find the electric field inside and outside a sphere of radius  $R$ , which carries a charge density proportional to the distance from the origin,  $\rho = kr$  (for some constant  $k$  and  $r < R$ ).

**Problem 4** (one part)

The Maxwell equation known as Faraday's Law of Induction has the form

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Discuss the importance of the minus sign in this equation. In other words, describe how the absence of this minus sign (keeping all other Maxwell equations the same) would affect electromagnetic phenomena, conservation laws of physics, etc.

**Problem 5** (three parts)

(a) State the variational principle of quantum mechanics, used to construct an approximate ground state wave function and energy for a given Hamiltonian  $H$ .

(b) Write down the Hamiltonian for a three-dimensional harmonic oscillator.

(c) Use the variational principle to estimate the ground state energy of the three-dimensional harmonic oscillator, using the trial wave function  $\psi(\mathbf{r}) = Ce^{-\gamma r}$ , where  $C$  is a normalization constant.

**Problem 6** (one part)

The specific heat capacities of liquid water and ice at atmospheric pressure and for several degrees Celsius below the freezing point are given by the equations

$$c_p(\text{water}) = 4222 - 22.6 T$$

$$c_p(\text{ice}) = 2112 + 7.5 T$$

in joules per kilogram-degree. Here  $T$  is the temperature in Celsius. What is the specific entropy difference between ice and water at -10 Celsius. The latent heat of fusion is 335200 J/kg.

## January 2006 Written Comprehensive Exam - Day 2

This is a closed-book, closed-note exam. You may use a calculator, but only for arithmetic functions (not for referring to notes stored in memory). Attempt all six problems. Start each problem on a new sheet of paper and use one side only. Print your name on each piece of paper that you submit. For full credit you must show your work and/or explain your answers.

**Problem 1** (one part)

You are on a ferris wheel, and your apparent weight at the top is 740 N and at the bottom it is 800 N. If the wheel completes one revolution every 35 s, what is the radius  $R$  of the ferris wheel?

**Problem 2** (three parts)

In the ride “Spindletop” at an amusement park, people stand against the inner wall of a hollow vertical cylinder of radius 3 m. The cylinder starts to rotate, and when it reaches a constant rotation rate of 0.5 rev/s, the floor on which the people are standing drops by about 0.5 m. The people remain pinned against the wall.

(a) Draw a force diagram for a person in this ride, after the floor has dropped.

(b) What minimum coefficient of static friction is required if the person in the ride is not to slide downward to the new position of the floor?

(c) Does your answer in (b) depend on the mass of the passenger?

**Problem 3** (one part)

A steady current  $I$  flows through a long cylindrical wire of radius  $a$ . The current is distributed in such a way that the current density  $j$  is proportional to  $r$ , the distance from the axis. Find the magnetic field, both inside and outside the wire.

**Problem 4** (two parts)

An annular cylinder of a substance with conductivity  $\sigma$  has a height  $h$ , an inner radius of  $r_1$ , and an outer radius of  $r_2$ . Electrodes are attached to the inner and outer surfaces so that current flows radially outward through the cylinder.

(a) Determine an expression for the total resistance of the cylinder in terms of the given variables.

(b) Show that your expression reduces to the familiar form for the resistance of a rectangular slab, in the limit of  $r_2 - r_1 \ll r_1$ .

**Problem 5** (three parts)

(a) A two-state quantum system is described by the Hamiltonian

$$H = \begin{pmatrix} a & b \\ b & a \end{pmatrix},$$

where  $a$  and  $b$  are real constants. Give a physical example of a system described by  $H$ , including a detailed explanation of the physical meaning of  $a$  and  $b$ .

(b) Find the eigenvalues of  $H$ .

(c) Find the normalized eigenstates of  $H$  for the special case of  $a = 0$ .

**Problem 6** (one part)

Calculate the maximum work in Joules obtainable from a heat reservoir consisting of 200 kg of iron heated initially to a temperature of 1500 C, using the ocean at 12 C as the second heat reservoir. Assume that the specific heat capacity of the iron is constant and equal to 0.60 J/gm deg.