Department of Physics and Astronomy **University of Georgia**

August 2006 Written Comprehensive Exam – Day 1

This is a closed-book, closed-note exam. You may use a calculator, but only for arithmetic functions (NOT for referring to notes stored in memory). Attempt all six problems. Start each problem on a new sheet of paper and use **one side only**. Print your name on each piece of paper that you submit. For full credit you must show your work and/or explain your answer.

Problem 1: (one part)

A ladder of length l and weight W_l has one end against a frictionless vertical wall and the other end on the horizontal ground. The ladder makes an angle α with the ground. What is the minimum coefficient of static friction between the ladder and the ground that will enable a person of weight W_p to climb to the top of the ladder without having it slip?

Problem 2: (two parts)

The Hamiltonian H for an axially symmetric rotator is given by:

$$H = \frac{1}{2I_1}\left(L_x^2 + L_y^2\right) + \frac{L_z^2}{2I_2}$$

L_x, L_y, and L_z are the operators representing the components of the standard orbital angular momentum vector. The constants, I_1 and I_2, are the moments of inertia of the rotator.

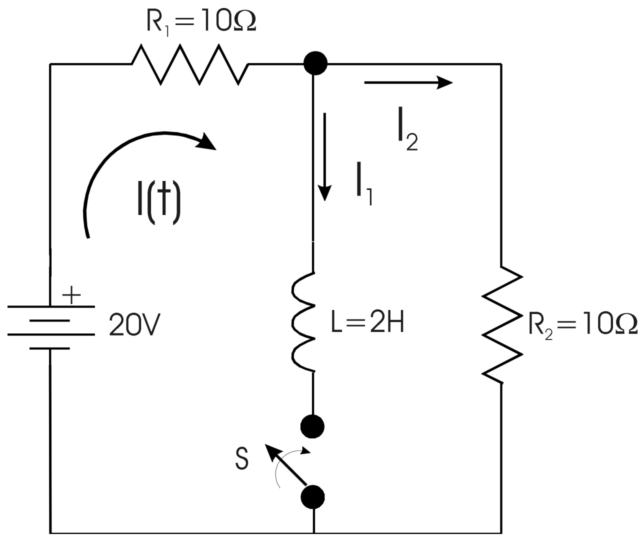
(a.) What are the eigenvalues of H?

(b.) Sketch the energy spectrum assuming $I_1 > I_2$.

(Hint: Express the above Hamiltonian in terms of those angular momentum operators whose eigenfunctions and eigenvalues you know!

Problem 3: (one part)

For the circuit below, find I(t) for t>0, i.e. after the switch S is closed at t=0. Hint: Use KCL (Kirchhoff's current law) $I(t)=I_1(t) + I_2(t)$ in setting up the KVL (Kirchhoff's voltage law) equations in terms of I, I_1, and I_2.



Problem 4: (one part)

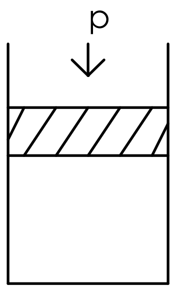
The Hamiltonian operator for a harmonic oscillator can be written as

$$H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 X^2 .$$

where X is the position and P is the momentum of the mass m. Rewrite this Hamiltonian in terms of the number operator $N = a^\dagger a$ where a and $a^\dagger$ are the creation and annihilation operators

$$a = \frac{1}{\sqrt{2}}\left(\hat{X} + i\hat{P}\right) \text{ and } a^\dagger = \frac{1}{\sqrt{2}}\left(\hat{X} - i\hat{P}\right), \text{ where } \hat{X} = \sqrt{\frac{m\omega}{\hbar}}X \text{ and } \hat{P} = \frac{1}{\sqrt{\hbar m\omega}}P$$

Problem 5: (3 parts)



One mole of *He* gas is in a cylinder which is held at constant pressure P = 1 atm. (*Note: 1 Pa. = 1* Nm^{-2} *= 0.99 x* 10^{-5} *atm.*)

(a.) The system has volume V = 50 liters. What is the temperature of the gas?

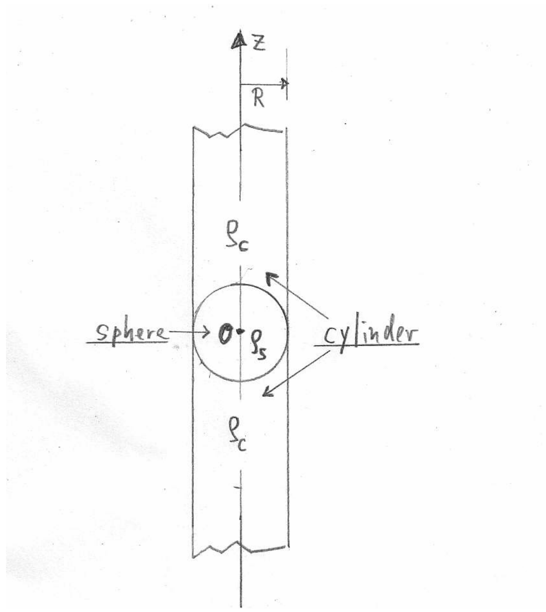
(b.) What is the enthalpy of the system?

(c.) Use kinetic theory to estimate the root mean square speed of the gas particles in the container.

Problem 6: (one part)

A spherical volume of radius R, filled with a homogeneous charge density ρ_S, is embedded in a concentric cylinder of the same radius R and infinite length, as shown in the drawing below. The interior of the cylinder, excluding the volume of the sphere, is filled with a homogeneous charge density ρ_C.

Find the electrical potential V(**r**) at any position **r** outside the cylinder, expressed in terms of ρ_C, ρ_S and spherical coordinates (r,θ,φ), with the center of the sphere as the coordinate origin and the central axis of the cylinder as the z-axis. Assume V(**r**) is set to V=0 at the center of the sphere, r=0.



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August 2006 Written Comprehensive Exam – Day 2

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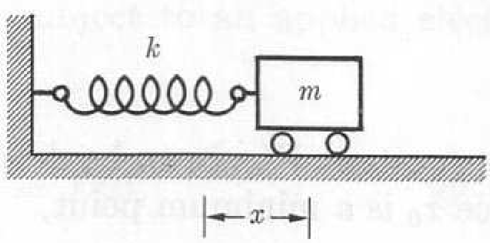
Problem 1: (one part)

For a free relativistic particle moving in one dimension with mass *m* and speed *v*, the total energy is $E = \gamma mc^2$ and the momentum is $p = \gamma mv$ with $\gamma = \left[1 - \left(v/c\right)^2\right]^{-1/2}$. For the quantum wave representing the particle, the group velocity is defined as $v_g = d\omega/dk$ where ω represents the angular frequency and k represents the wavevector of the quantum wave. Calculate the group velocity of the wave and compare it with the speed of the particle.

Problem 2: (three parts)



Consider a one-dimensional oscillator with mass *m* and spring constant *k* subject to a damping force. The equation of motion for this system is:

$$m\ddot{x} = -kx - \lambda\dot{x} \ .$$

(a.) How does the position *x* vary as a function of time in terms of m, k, and λ ?

(b.) How does the frequency of oscillation compare with that of an undamped oscillator?

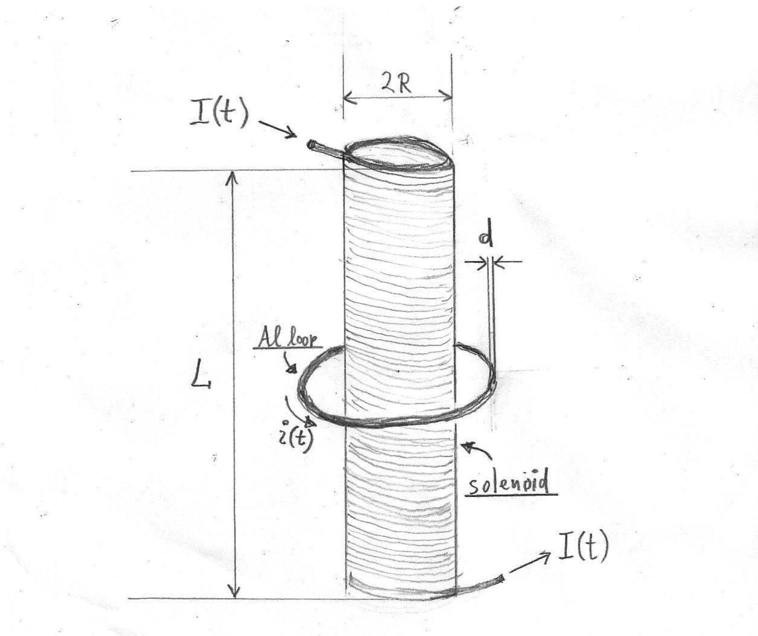
(c.) If the damping is small, how does the total energy of the oscillator vary with time?

Problem 3: (one part)

A very thin copper wire has been tightly wound into a very tall, thin cylindrical solenoid of height L=1240cm, with N=180,000 circular turns of radius R=6.0cm. An aluminum (*Al*) wire of length C=50cm and wire diameter d=0.02cm has been bent into a closed, conducting circular loop, encircling the solenoid concentrically, as shown in the drawing below. The electrical resistivity of *Al* is $\rho = 2.75 \times 10^{-8}$ Ohm•meter.

A 25Hz alternating electrical current $I(t) = I_0 \sin(\omega t)$ with I_0=370mA is flowing through the solenoid. The magnetic field **B** due to I(t) is approximately homogeneous inside the solenoid, with $|\mathbf{B}|=\mu_0 |I| N/L$ and $\mu_0=4\pi \times 10^{-7}$ Tm/A; and $|\mathbf{B}|$ is negligible outside the solenoid. Neglect the contribution to the **B**-field from the induced current in the *Al* loop.

What is the magnitude |i(t)| of the induced electrical current i(t) flowing through the *Al* loop at time t_1=35.0ms ?



Problem 4: (one part)

Given the mass and radius of the Earth as M_E and R_E, respectively, find an expression for the escape velocity of a particle of mass m in the absence of air resistance. "Escape" here means that the particle leaves the surface of the Earth vertically, i.e. in a radial direction away from the Earth's center, with initial speed v_i and reaches a stationary point located infinitely far from Earth.

Problem 5: (two parts)

A non-relativistic particle, propagating to the right, encounters a potential wall at x=0 given by

$$V(x) = \frac{\hbar^2}{m}\Omega\delta(x)$$

where $\delta(x)$ is the Dirac delta function.

(a) Solve the time-independent 1D Schrödinger equation for this potential, and find the forward and backward scattering amplitudes in terms of Ω. (Hint: consider what are the appropriate boundary conditions for the wavefunction at x=0.)

(b) Show that for $\Omega \to \infty$ (strong potential), the particle is totally reflected, and that for $\Omega \to 0$ (weak potential), the backward scattering amplitude is inversely proportional to the particle wavenumber.

Problem 6: (one part)

Monochromatic light from a helium-neon laser (λ=632.8 nm) is incident normally on a diffraction grating ruled at 6000 grooves/cm. Find the angles of all observed interference maxima measured from the direction of incidence.