Department of Physics and Astronomy **University of Georgia**

August 2005 Written Comprehensive Exam - Day 1

This is a closed-book, closed-note exam. You may use a calculator, but only for arithmetic functions (not for referring to notes stored in memory). Attempt all six problems. Start each problem on a new sheet of paper and use one side only. Print your name on each piece of paper that you submit. For full credit you must show your work and/or explain your answers.

Problem 1 (three parts)

A satellite of mass 950 kg is being towed by a spaceship in empty space. The two vessels are connected by a uniform rope 50 meters in length whose mass per unit length is 1 kg/m. The spaceship is accelerating in a straight line with acceleration 5 m/sec^2.

(a) What is the magnitude of the force exerted by the spaceship on the rope?

(b) Calculate the tension along the rope.

(c) Due to an equipment mishap, the spaceship instantaneously changes its acceleration to a deceleration of 1 m/sec^2. Describe in detail the consequences of this mishap (be quantitative).

Problem 2

An 8 g bullet is fired into a 2.5 kg block that is initially at rest at the edge of a frictionless table of height 1 m, pushing the block over the edge. The bullet remains in the block, and after impact the block lands 2 m from the bottom of the table. Determine the initial speed of the bullet.

Problem 3 (four parts)

Unpolarized light propagating in the $+\hat{\mathbf{z}}$ direction is incident on a set of $n+1$ perfect linear polarizers situated along the z axis. The first polarizer's transmission axis is parallel to the x axis, and each successive polarizer's transmission axis is rotated by an angle ϕ/n in the xy-plane, relative to the previous polarizer.

(a) Describe the light's polarization direction after it exits the last polarizer, and give an expression for the light's final intensity I in terms of its initial intensity I_0 and the other relevant quantities.

(b) Determine the first-order approximation to the final intensity for large n.

(c) Now assume the polarizers are not ideal: although each polarizer still transmits light that is perfectly linearly polarized, the transmitted intensity is reduced by some small fraction ϵ relative to an ideal polarizer. How does this change your expression for the final intensity from part (a)?

(d) With the non-ideal polarizers of part (c), there is some finite value of n for which the final intensity is a maximum. Derive an approximate expression for this $n_{\max}$, assuming that n is large and ϵ is small. (Hint: You may find it easier to maximize $\ln I$ instead.)

Problem 4

A solid sphere of radius a is homogeneously charged, with positive charge density ρ. The sphere is located at the origin of the coordinate system. Into this sphere we cut 6 spherical holes of diameter $a/4$ located on the three Cartesian coordinate axes at distances $\pm a/2$ from the center of the original sphere. Calculate both the potential and the electric field (magnitude and direction) at the point $(2a, 0, 0)$.

Problem 5

Calculate the minimum amount of work (in joules) required to freeze one liter of water originally at temperature $T = 20°\text{C}$. Assume the heat reservoir to be at $20°\text{C}$. The heat of fusion of water is $80\,\text{cal/gm}$.

Problem 6 (three parts)

A particle in free space is prepared initially in a wave packet described by

$$\psi(x) = \left(\frac{1}{2\pi a^2}\right)^{\frac{1}{4}} e^{-x^2/2a^2}.$$

(a) What is the probability that its momentum is in the range between p and $p + dp$?

(b) What is the expectation value of the energy?

(c) Give a rough estimate, based on the size of the wave function and the uncertainty principle, for why the answer to (b) should be roughly what it is.

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August 2005 Written Comprehensive Exam - Day 2

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Problem 1

A planet of mass m is orbiting a star of mass M. The planet experiences a small drag force $\mathbf{F} = -\alpha\mathbf{v}$ due to motion through the star's dense atmosphere. Assuming an essentially circular orbit with radius $r = r_0$ at $t = 0$, calculate the time dependence of the radius.

Problem 2 (three parts)

A horizontal string is attached to a 0.25 kg mass lying on a rough, horizontal table. The string passes over a light, frictionless pulley, and a 0.4 kg mass is then attached to its free end. The coefficient of sliding friction between the 0.25 kg mass and the table is 0.20. The masses of the string and pulley may be neglected.

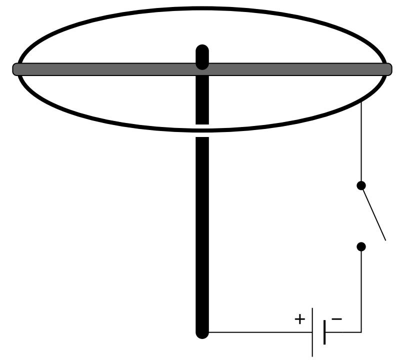
(a) Using the work–kinetic energy theorem, determine the speed of the masses after each has moved 20 m from rest.

(b) Determine the mass that must be added to the 0.25 kg mass so that, given an initial velocity, the masses continue to move at a constant speed.

(c) What mass must be removed from the 0.4 kg mass so that the same outcome as in part (b) is achieved?

Problem 3 (two parts)

A horizontal conducting bar of mass m and length ℓ is mounted to a vertical frictionless conducting axle. The outer edge of the bar makes contact with a circular frictionless conducting rail. The rail and axle are connected to a constant voltage source $\mathcal{E}$ on a switch. Each half of the bar has a resistance R, but all other conductors have a negligible resistance. The entire setup sits at rest in a uniform magnetic field $-B\hat{\mathbf{z}}$.



(a) At time $t = 0$, the switch is closed. Find the initial magnetic torque on the bar.

(b) Determine the equation of motion describing the bar's angular speed ω. Solve this differential equation to find $\omega(t)$.

Problem 4

A long cylindrical capacitor has an inner conductor of radius 1 cm and an outer one of radius 2 cm. The space between the conductors is filled with a material of (relative) dielectric constant $\epsilon = 4$ and a breakdown strength of 2×10^7 V/m. Calculate the voltage across the capacitor at breakdown.

Problem 5

A solid of density ρ_1 melts at pressure P and absolute temperature T to form a liquid of density ρ_2. The latent heat of melting per gram of solid is L. Find the change of entropy ΔS and the change of internal energy ΔU resulting from the melting of one gram of the solid.

Problem 6

Show that for a quantum mechanical system described by the Hamiltonian

$$H = \frac{p^2}{2m},$$

the probability flux

$$\mathbf{j} = \frac{\hbar}{2im}\left[\psi^* \boldsymbol{\nabla} \psi - (\boldsymbol{\nabla} \psi^*) \psi\right]$$

satisfies the continuity equation

$$\frac{\partial}{\partial t}\psi^* \psi + \boldsymbol{\nabla} \cdot \mathbf{j} = 0.$$