STATIC EQUILIBRIUM

I. PURPOSE

In this experiment you will apply the concept of **torque** and learn what we mean when we say that a body is in **static equilibrium**. You will examine an object in static equilibrium and determine some unknown forces acting on the object both analytically and experimentally.

II. THEORY

When forces are acting on a body the conditions for translational equilibrium are:

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0,$$

and the condition for rotational equilibrium about an axis of rotation is:

$$\sum \tau = 0$$

$\boldsymbol{\tau}$ is the torque and is defined as $\boldsymbol{\tau} = \mathbf{F}*r*\sin\theta$, where **F** is the force acting on a body at some angle θ at some distance r from the axis of rotation, causing it to rotate around that axis. r*sinθ is the **moment arm**, or the perpendicular distance from the **line of action** of the force to the axis of rotation.

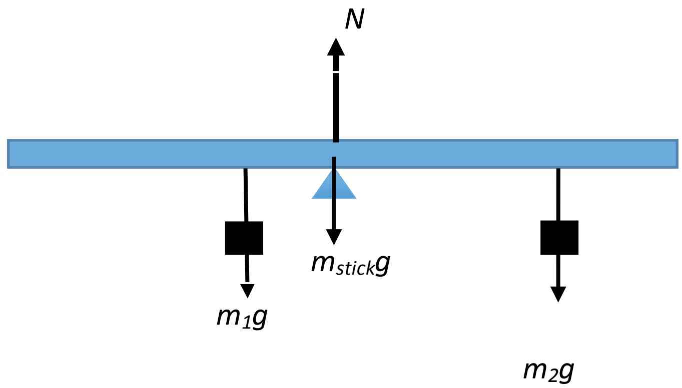
*When an object is in translational equilibrium and rotational equilibrium, we say it is in **static equilibrium.***

Earlier this semester you performed an equilibrium experiment for the case when all of the forces are acting on the same point. This was the lab with the force table called: "Equilibrium of a Point Mass". In that lab since all of the forces were acting on a single point (two-dimensionally) you only needed $\sum F_x = 0$ and $\sum F_y = 0$ to solve.

In most real world structures the forces acting on the structure will be acting at various locations of the structure. In these cases if there is more than one unknown variable then we will need to use $\sum \tau = 0$ to help solve for our first unknown. By choosing to sum the torques acting at one of the unknown force locations you will be able to create an equation that does not involve that unknown force (since it doesn't cause a torque at its point of contact).

Consider the following example. A meter stick with a mass of 100 grams is placed on a fulcrum which is at its center of mass. A total mass of 100 grams (m_1) hangs from it at the 40 cm mark. What total mass do you need to place at the 70 cm mark in order for the system to be in static equilibrium?

Free-body diagram:



Since there are only forces in the y-direction

$$\sum F_y = 0$$

therefore, $\mathbf{N} - m_1g - m_{stick}g - m_2g = 0$

Since both **N** and m_2g are unknown it helps to take the axis of rotation at one of these points. If we pick the center of mass of the stick, then both **N** and $m_{stick}g$ have no torques at this point and

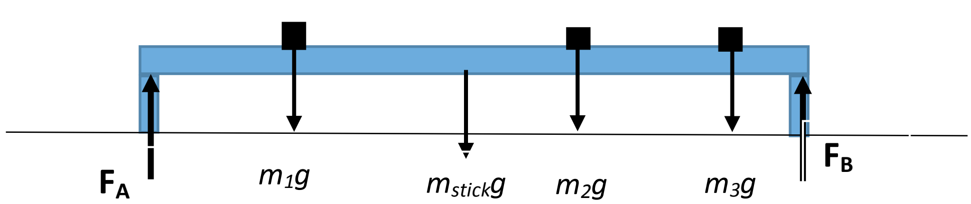
$$\sum \tau = 0$$

gives us (taking counterclockwise torques to be positive and clockwise torques to be negative):

$(m_1g)(r_1) - (m_2g)(r_2) = 0$, so $m_2 = (m_1)(r_1)/(r_2) =$ 50 grams.

III. APPARATUS

Today in lab a meter stick resting on two force sensors will serve as our structure (this is similar to a bridge). Loads (weights) will be placed at various locations along the meter stick and your job will be to predict how much force will be required at the supports. You will then put your results to the test by seeing if force sensors agree with your predictions.



IV. PROCEDURE

Analytical

1. Measure the mass of the meter stick and record it on the worksheet for trials 1 and 2.
2. Predict the location of the center of mass of the meter stick.
3. Using the triangular block of wood, balance the meter stick on its edge to determine the actual location of the center of mass. Record this value on the worksheet and answer the questions regarding this procedure.
4. With the meter stick balancing on the triangular block at the center of mass, have your partner hold the meter stick level. Add a 20g mass at the 20cm mark and place a 50g mass on the other side of the center of mass of the meter stick. Experiment with the location of the 50g mass in order to find where this added mass will restore balance to the system. Record the location of this mass and discuss these results.
5. Record the values of the masses and their locations given to you by your TA for each trial. **IMPORTANT NOTE**: in the values given the TA will include the masses of the hangers you will use when you perform the experiment so when you do the experiment you need to simply add the appropriate mass to the mass hangers to match the total given. (The mass of the modified paper clip hanger is approximately 1 gram.)
6. Referring to the diagram in the APPARATUS section, draw a free-body diagram for each trial in the space provided.
7. Solve for the unknown values of the forces at the supports for each trial. These will be the **calculated** values of $\mathbf{F_A}$ and $\mathbf{F_B}$. Show your calculations in the spaces provided and indicate where you chose your axis of rotation.

Experimental

1. Log in to the computer and initiate the software.
2. You should see readouts for the two force sensors. (The sensors should be set to the 10N setting.) Notice that the sensors are not reading zero. This is because of the weight of the screw and washers placed on the sensors to help balance the meter stick.
3. Zero the force sensors. From the **Menu** choose **Experiment** and click on **Zero.** In the window that appears you should see both detectors selected, if they are not, select them. Click **OK**.
4. Now place the meter stick on edge (with the metric scale facing you) so that one force sensor is at the 10 cm mark and the other is at the 90 cm mark. As you add and move masses on the meter stick keep checking to see that the sensors are at these marks.
5. Place the appropriate number of modified paper clips on the meter stick to the location or locations specified. Hang a mass hanger from the clip and add the mass or masses needed to get the given total mass at each location.
6. Record the values of the forces of the force sensors for each trial. These are the **experimental** values of $\mathbf{F_A}$ and $\mathbf{F_B}$.
7. Calculate the %Errors.

Name:_____________________________ Day/Time___________________

Lab Partner:____________________________ Instructor___________________

STATIC EQUILIBRIUM

Center of Mass

Predicted Center of Mass _______ Experimental Center of Mass_______

1. Explain in your own words what is meant by the center of mass.

2. Did your prediction for the center of mass location match the experimental result? If no, what might be some reasons for the difference?

Balancing the 20g Mass

3. Location of 50 g mass________
4. Discuss the results:

Trial 1:

Free-body diagram (label everything)

mass 1:_______________ location 1:_______________

mass of meter stick:__________ Center of Mass location:____________

Calculations for F_A and F_B:

Calculated F_A:_________________ Calculated F_B:_________________

Experimental F_A:_________________ Experimental F_B:_________________

% Error F_A:_________________ % Error F_B:_________________

<u>Trial 2:</u>

Free-body diagram (label everything)

mass 1:_______________ location 1:_______________

mass 2:_______________ location 2:_______________

mass of meter stick:__________ Center of Mass location:____________

<u>Calculations for F_A and F_B:</u>

Calculated F_A:_________________ Calculated F_B:_________________

Experimental F_A:_________________ Experimental F_B:_________________

% Error F_A:_________________ % Error F_B:___________

DISCUSSION OF RESULTS

5. How did your calculated values in trials 1 and 2 compare to your experimental values? What were some possible sources of error?

Additional Questions:

6. Using the set up of the last part of the experiment having two weights on the meter stick, place the imaginary axis of rotation at one end of the meter stick.
   a. Set up the two equations of the sum of the forces and torques using the end of the meter stick as the axis of rotation. DO NOT SOLVE.

   b. Is there a point on the meter stick where the sum of the torques would not be zero, that is, is there a location on the meter stick where the system is NOT in static equilibrium? Explain.

7. Name at least two situations you have encountered outside of the lab that involve torque and describe how they illustrate it.