1. **Cross/Outer product**

Given a magnetic moment $\overline{\mu} = \mu_x \hat{x} + \mu_y \hat{y} + \mu_z \hat{z}$ or $\overline{\mu} = (\mu_x, \mu_y, \mu_z)$, and a magnetic field $\overline{B} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$ or $\overline{B} = (B_x, B_y, B_z)$, the cross product of $\overline{\mu}$ and $\overline{B}$ can be written as,

$$\overline{\mu} \times \overline{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \mu_x & \mu_y & \mu_z \\ B_x & B_y & B_z \end{vmatrix}$$
$$= \left(\mu_y B_z - B_y \mu_z\right)\hat{x} + \left(B_x \mu_z - \mu_x B_z\right)\hat{y} + \left(\mu_x B_y - B_x \mu_y\right)\hat{z}$$

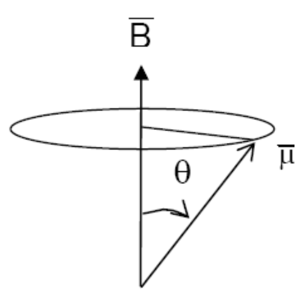
If $\overline{B} = B_0 \hat{z}$, then

$$\overline{\mu} \times \overline{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \mu_x & \mu_y & \mu_z \\ 0 & 0 & B_0 \end{vmatrix}$$
$$= \left(\mu_y B_0\right)\hat{x} - \left(\mu_x B_0\right)\hat{y} = \left(\mu_y \hat{x} - \mu_x \hat{y}\right) B_0$$

The above results indicate that for a magnetic field oriented along the z-axis, only the x and y component of $\overline{\mu}$ are effected and result in a vector in the *xy*-plane. This can be appreciated by calculating the magnitude of the resulting vector

$$\left|\overline{\mu} \times \overline{B}\right| = \sqrt{\left(\mu^2{}_x + \mu^2{}_y\right)} B_0$$
$$= \left|\overline{\mu}\right| B_0 \sin\theta$$
$$= \mu_{xy} B_z$$

Notice that $\mu_{xy} = \left|\overline{\mu}\right| \sin\theta$ is the projection of $\overline{\mu}$ into a plane orthogonal to $\overline{B}$, which is the *xy*-plane. Also notice that in the above equation, the x-component is rotated in the y-direction and y-component is rotated into the x-direction. Therefore the resulting vector is perpendicular to both $\overline{\mu}$ and $\overline{B}$ directions.



2. **Dot/Inner product**

An inner product is a generalization of the dot product. In a vector space, it is a way to multiply vectors together, with the result of this multiplication being a scalar.

More precisely, for a real vector space, an inner product $\langle \cdot, \cdot \rangle$ satisfies the following four properties. Let *u, v* and *w* be vectors and α be a scalar, then:

1. $\langle u+v, w\rangle = \langle u, w\rangle + \langle v, w\rangle$.

2. $\langle \alpha v, w\rangle = \alpha \langle v, w\rangle$.

3. $\langle v, w\rangle = \langle w, v\rangle$.

4. $\langle v, v\rangle \geq 0$ and equal if and only if $v = 0$.

Examples of inner product spaces include:

1. The real number $\mathbb{R}$, where the inner product is given by

$$\langle x, y\rangle = xy \qquad or \qquad x \cdot y = xy$$

2. The vector space $\mathbb{R}^n$, where the inner product is given by the dot product

$For\ \overline{X} = (x_1, x_2, ..., x_n)$ and $\overline{Y} = (y_1, y_2, ..., y_n)$

$$\begin{aligned}\langle \overline{X}, \overline{Y}\rangle = \overline{X} \cdot \overline{Y} &= (x_1, x_2, ..., x_n) \cdot (y_1, y_2, ..., y_n) \\ &= x_1 y_1 + x_2 y_2 + ... + x_n y_n\end{aligned}$$