Physics 1111
Problem Set #4 Solutions

Problem 1: Choose whether each of the following statements is true or false: (a) An object can be moving in one direction while the net force acting on it is in another direction. (b) An object at rest has no forces acting on it. (c) In order to move a massive crate sitting on the floor, the force you apply to the crate has to overcome the force of the crate pushing back on you. (d) An object can be in motion and still have zero net force acting on it.

(a) $\boxed{\text{True.}}$ For example, a car that is braking is moving forward, while the net force acting on it points backwards.

(b) $\boxed{\text{False.}}$ An object at rest has no *net* force acting on it, but there could be several forces acting on the object that cancel out.

(c) $\boxed{\text{False.}}$ Newton's Third Law guarantees that the force you apply to the crate is exactly equal and opposite to the force that the crate exerts back on you.

(d) $\boxed{\text{True.}}$ An object can be moving at constant velocity, in which case no net force acts on it according to Newton's First Law.

Problem 2: A bird cage, with a parrot inside, hangs from a scale. The parrot decides to hop from one perch to another (without flapping its wings). The total weight of the parrot and cage is 50 N. For the following questions, assume that the scale responds rapidly enough that it gives an accurate reading at every instant. (a) What is the reading on the scale before the parrot jumps? (b) What is the reading on the scale as the parrot starts its jump? (c) What is the reading on the scale when the parrot is in the air? (d) What is the reading on the scale as the parrot lands on the other perch?

(a) If the parrot and the cage are not moving at all, then the normal force on the system will equal its weight. Thus the scale will read $\boxed{\text{50 N.}}$

(b) In order to jump off the perch, the parrot needs to be accelerated upward, which means that the parrot must experience a net upward force. This is provided by the normal force. (This may sound kind of strange, since we're so used to thinking of the parrot's *legs* as providing the force. But in actuality, the legs push against the perch, and the perch, by Newton's Third Law, pushes back on the parrot. It's this reaction force that moves the parrot.)

Since the parrot requires a net upward force, the normal force must be greater than the weight. So the scale (which measures normal force) will read $\boxed{\text{more than 50 N.}}$

(c) While the parrot is in the air, the scale will read only the normal force required to oppose the perch's weight. So the force in this case is less than 50 N.

(d) When the parrot lands, it must once again be *accelerated upward*, because it has a downward velocity that must be brought to zero. Once again, the normal force acting on the parrot is what provides the net upward force required to "decelerate" it. So, the scale reading is more than 50 N.

Problem 3: You jump out of an airplane, and your parachute opens after a brief period of free fall. In order to slow your fall, the parachute must exert a force that is...

Before the parachute opens, you are falling downward and being accelerated by the force of gravity. If you want to slow your fall, then the *net* force on you has to be upward, in the opposite direction to your motion. This means that the force exerted by the parachute has to be greater than your weight.

Problem 4: You stand on the seat of a chair and then hop off. (a) During the time you are in flight down to the floor, the Earth is lurching up toward you. What is the approximate magnitude of the Earth's acceleration? (b) The Earth moves up through a distance of what order of magnitude?

(a) The Earth and I exert equal and opposite gravitational forces on each other by Newton's Third Law. My acceleration toward the Earth is $g = 9.80 \text{ m/s}^2$, my mass is about 65 kg, and the mass of the Earth is approximately 6×10^{24} kg. Since $m_{me}g = F_g = m_{Earth}a_{Earth}$, we get that $\boxed{a_{Earth} \sim 10^{-22} \text{ m/s}^2.}$

(b) The distance I travel is given by $d_{me} = gt^2/2$, and similarly the distance the Earth travels in the same time is $d_{Earth} = a_{Earth}t^2/2$. Divide one equation by the other to eliminate the time terms:

$$\frac{d_{Earth}}{d_{me}} = \frac{a_{Earth}}{g}$$

Since the distance I travel from chair to ground is about 0.5 m, $\boxed{d_{Earth} \sim 5 \times 10^{-24} \text{ m.}}$ This is several orders of magnitude smaller than an atom.

(This means, contrary to urban legend, that if everyone in China (or India, or whatever) jumped off chairs simultaneously, they would *not* significantly jolt the world.)

Problem 5: A 747 jetliner lands and begins to slow to a stop as it moves along the runway. (a) If its mass is 3.17×10^5 kg, its speed is 27.1 m/s, and the net braking force is 4.40×10^5 N, what is its speed 7.81 s later? (b) How far has it traveled in this time?

(a) This problem is a combination of kinematics and forces. First, we can use the net force and the plane's mass to calculate its acceleration, according to Newton's Second Law:

$$\sum F = ma \quad \Longrightarrow \quad a = \frac{F}{m} = \frac{-4.40 \times 10^5 \text{ N}}{3.17 \times 10^5 \text{ kg}} = -1.388 \text{ m/s}^2.$$

We write the braking force and the acceleration as negative because they are in a direction opposite to the plane's motion. Now, use the acceleration to compute the change in speed:

$$v_f = v_i + at = (27.1 \text{ m/s}) - (1.388 \text{ m/s}^2)(7.81 \text{ s}) = 16.26 \text{ m/s} \doteq \boxed{16.3 \text{ m/s.}}$$

(b) There are a couple of ways to solve this problem; here's one:

$$\Delta x = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(27.1 + 16.26)(7.81) = \boxed{169 \text{ m.}}$$

Problem 6: After falling from rest at a height of 33.1 m, a 0.442 kg ball rebounds upward, reaching a height of 22.1 m. If the contact between ball and ground lasted 2.35 ms, what average force was exerted on the ball?

If we wanted to think of this problem in terms of events, there are four instants in time that are important here. The first is when the ball starts from rest, the second is right before the ball hits the ground. Events 3 and 4 are right *after* the ball hits the ground, and when it reaches its maximum height. Between the first pair and last pair of events, the ball is in free fall (just gravity acts on it). This means, in particular, that we can find the ball's velocity just before it hits the ground, and just after it hits (v_2 and v_3):

$$v_f^2 = v_i^2 + 2a(\Delta x) \quad \Longrightarrow \quad v_2^2 = 0 - 2g(x_2 - x_1) \quad \text{and} \quad 0 = v_3^2 - 2g(x_4 - x_3).$$

With the numbers we're given, we obtain

$$\begin{aligned} v_2^2 &= 2(9.81)(33.1) & \qquad v_3^2 &= 2(9.81)(22.1) \\ v_2 &= -25.484 \text{ m/s} & \qquad v_3 &= 20.823 \text{ m/s}. \end{aligned}$$

Notice that I've written v_2 as a *negative* number to reflect the fact that the ball is traveling downward before it hits the ground. (This negative sign isn't frivolous! Remember, when taking the square root of a quantity, the answer can be positive or negative.)

The time interval between events 2 and 3 is when the ball is in contact with the ground. Knowing the initial and final velocities for this interval and the elapsed time, we can find the average acceleration:

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_3 - v_2}{\Delta t}.$$

By Newton's Second Law, this acceleration is related to the average net force exerted on the ball:

$$\overline{F}_{net} = m\overline{a} = m\frac{v_3 - v_2}{\Delta t}.$$

Putting in the numbers,

$$\overline{F}_{net} = (0.442 \text{ kg})\frac{(20.823 \text{ m/s}) - (-25.484 \text{ m/s})}{2.35 \times 10^{-3} \text{ s}} = \boxed{8.71 \text{ kN.}}$$

That's a large force!

Problem 7: Your groceries are in a bag with paper handles. The handles will tear off if a force greater than *50.5* N is applied to them. (a) What is the greatest mass of groceries that can be lifted safely with this bag, given that the bag is raised with constant speed? (b) What is the greatest mass of groceries that can be lifted safely with this bag, given that the bag is raised with an acceleration of *1.30* m/s^2?

(a) First of all, the forces acting on the bag are its weight mg downward, and the upward force on the handles, which we'll call F. There are no horizontal forces to worry about, so the net force on the bag can be written as

$$\sum F_y = F - mg.$$

If the bag is raised with constant speed, then it is not accelerated, so by Newton's Second Law the net force on it must be zero. So $F = mg$, and since we know $F_{max} =$ 50.5 N we can calculate $m_{max} = F_{max}/g = \boxed{5.15 \text{ kg.}}$

(b) The algebraic expression for the net force ($\sum F_y = F - mg$) is unchanged from part (a). Now however the net force isn't zero, because the bag is being accelerated upwards. So by Newton's Second Law we relate motion to forces by $\sum F_y = ma$. Apply this to the expression for net force above, and solve for m to get:

$$ma = \sum F_y = F - mg \qquad \Longrightarrow \qquad m = \frac{F}{a + g}$$

Then plugging in the numbers, $\boxed{m = 4.55 \text{ kg.}}$ It should make sense to you that this mass is less than in part (a).

Problem 8: When you lift a bowling ball with a force of *81.6* N, the ball accelerates upward with an acceleration a. If you lift with a force of *93.3* N, the ball's acceleration is $2.01a$. (a) Calculate the weight of the bowling ball. (b) Calculate the acceleration a.

The key to this problem is realizing that the force that you apply to the ball is *not* the net force, and therefore isn't directly proportional to the ball's acceleration. The ball also experiences a gravitational force.

In the first case, when you lift the ball with a force F_1, the net vertical force on the ball is $F_1 - mg$ (lifting force F_1 upward, and weight downward), and its acceleration is a. When you lift the ball with a new force F_2, the net force is $F_2 - mg$ and its acceleration is some multiple of a; let's call it c. So we have the two equations

$$F_1 - mg = ma$$
$$F_2 - mg = mca.$$

(a) Let's multiply the first equation by c, so that we can set it equal to the second equation:

$$c(F_1 - mg) = mca = F_2 - mg.$$

Now we can rearrange to solve for the weight of the bowling ball:

$$cF_1 - F_2 = cmg - mg \quad \Longrightarrow \quad (c-1)mg = cF_1 - F_2 \quad \Longrightarrow \quad mg = \frac{cF_1 - F_2}{c-1}.$$

Putting in the numbers given,

$$mg = \frac{2.01(81.6 \text{ N}) - (93.3 \text{ N})}{2.01 - 1} = \frac{70.716 \text{ N}}{1.01} = \boxed{70.0 \text{ N.}}$$

(b) Using the result of part (a), we solve for the acceleration:

$$ma = F_1 - mg \quad \Longrightarrow \quad a = \frac{F_1 - mg}{m} = \left(\frac{F_1 - mg}{mg}\right) g.$$

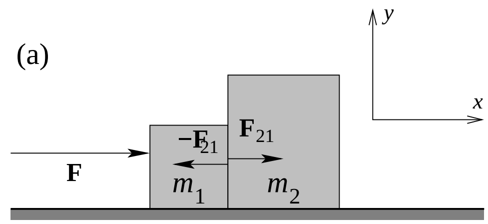
With this little trick of multiplying by g/g, we can find the acceleration *without* explicitly solving for the bowling ball mass!

$$a = \frac{81.6 \text{ N} - 70.0 \text{ N}}{70.0 \text{ N}}(9.81 \text{ m/s}^2) = \boxed{1.62 \text{ m/s}^2.}$$

Problem 9: Two boxes sit side-by-side on a smooth horizontal surface. The lighter box has a mass of *5.44* kg, the heavier box has a mass of *7.77* kg. (a) Calculate the contact force between these boxes when a horizontal force of *5.03* N is applied to the light box. (b) Calculate the contact force if the 5.03 N force is applied to the heavy box instead.

There are a couple of ways to solve this problem. We could draw a free-body diagram for each box, labeling the forces on each of them, and then we can apply Newton's Second Law for each box. So for example, in part (a), we would write for the small box (m_1):

(a)

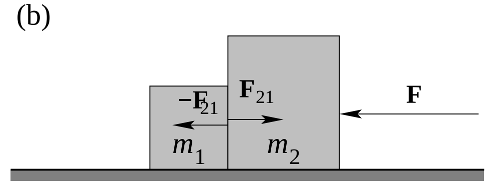


$$\sum F_{1x} = F - F_{21} = m_1 a$$

and for the large box:

$$\sum F_{2x} = F_{21} = m_2 a.$$

(b)



We have two equations for two unknown quantities (the acceleration a and the contact force F_{21}), which we can solve. (We don't particularly care about the vertical forces, since the motion is entirely horizontal.)

Alternatively, let's first consider the two boxes to be one object, since they move together. Then the net force is just our applied force F, and the total mass is $m_1 + m_2$, so we can calculate the acceleration:

$$\sum F_x = F = (m_1 + m_2)a \quad \Longrightarrow \quad a = \frac{F}{m_1 + m_2}.$$

Putting in the numbers yields

$$a = \frac{5.03 \text{ N}}{(5.44 \text{ kg}) + (7.77 \text{ kg})} = 0.3808 \text{ m/s}^2.$$

The acceleration will be the same magnitude in both parts of the problem.

(a) The net force on the heavy box is just the force of contact due to the light box pushing on it. So

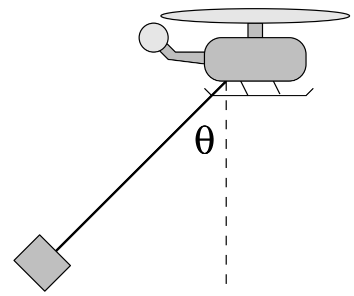
$$F_{21} = m_2 a = (7.77 \text{ kg})(0.3808 \text{ m/s}^2) = \boxed{2.96 \text{ N.}}$$

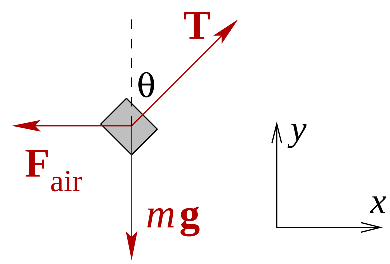
(b) In this case, the net force on the *light* box is the force of contact:

$$F_{21} = m_1 a = (5.44 \text{ kg})(0.3808 \text{ m/s}^2) = \boxed{2.07 \text{ N.}}$$

Problem 10: A fire helicopter carries a *612* kg bucket of water at the end of a *22.5* m long cable. Flying back from a fire at a constant speed of *41.5* m/s, the cable makes an angle of *45.2*° with respect to the vertical. Determine the force of air resistance on the bucket.

The free-body diagram for the helicopter is shown at right. The force of air resistance on the bucket acts directly opposite to the bucket's motion. Breaking this diagram down into components yields the following sums of forces:





$$\sum F_x = T\sin\theta - F_{air}$$
$$\sum F_y = T\cos\theta - mg.$$

Because the bucket is moving at a constant velocity, its acceleration is zero. Thus, the forces in each direction have to sum to zero, by Newton's Second Law:

$$T\sin\theta - F_{air} = 0 \implies F_{air} = T\sin\theta$$
$$T\cos\theta - mg = 0 \implies T\cos\theta = mg.$$

By solving the second equation for T, and plugging this into the first equation, we can get an answer for the air resistance:

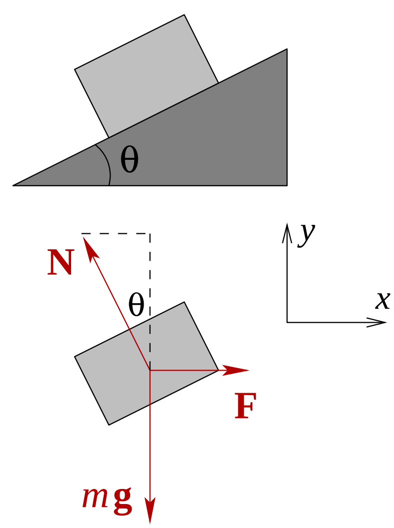
$$T = \frac{mg}{\cos\theta} \implies F_{air} = T\sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta.$$

Plugging in the numbers yields

$$F_{air} = (612)(9.81)\tan 45.2^\circ = \boxed{6.05\text{ kN.}}$$

Problem 11: A *60* kg crate is pushed at constant speed up the frictionless *24*° ramp shown in the figure. (a) What horizontal force F is required? (b) What force is exerted by the ramp on the crate?

This problem is quite similar to the "Sisyphus" example in class, except in this case the pushing force is purely horizontal, instead of being directed up the ramp. Because there's no acceleration (the crate is moving at constant velocity), it actually turns out to be easiest to choose a standard horizontal/vertical coordinate system, as opposed to one oriented along the ramp.



Based on this choice of coordinate system, we need to resolve the normal force into x and y components. The angle of the ramp is the same as the angle of the normal force from vertical, so we can write our sums of forces as

$$\sum F_x = F - N\sin\theta$$
$$\sum F_y = N\cos\theta - mg.$$

Now we apply what we know about the motion, namely that $a_x = a_y = 0$. By Newton's Second Law,

$$\sum F_x = ma_x \quad \Longrightarrow \quad F - N\sin\theta = 0$$
$$\sum F_y = ma_y \quad \Longrightarrow \quad N\cos\theta - mg = 0.$$

The second of these equations can be solved for the normal force:

$$N\cos\theta = mg \quad \Longrightarrow \quad N = \frac{mg}{\cos\theta}.$$

And then we can put this expression into the first equation to solve for F:

$$F = N\sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta.$$

Finally, let's put in the numbers. The horizontal force is

$$F = (60)(9.81)\tan 24^\circ = \boxed{262\text{ N}}$$

and the normal force is

$$N = \frac{(60)(9.81)}{\cos 24^\circ} = \boxed{644\text{ N.}}$$