Supplemental Material for “Catch-Disperse-Release Readout for Superconducting Qubits”

Eyob A. Sete¹, Andrei Galiautdinov¹,², Eric Mlinar¹, John M. Martinis³, and Alexander N. Korotkov¹

¹University of California, Riverside, California 92521, USA
²University of Georgia, Athens, Georgia 30602, USA
³University of California, Santa Barbara, California 93106, USA
Evolution of Wigner function and probability distributions

In this supplemental material, we present the evolution of the Wigner function for resonator field corresponding to initial states $|00\rangle$ and $|10\rangle$. Here the eigenstate $|10\rangle = \cos \theta_1 |00\rangle - \sin \theta_1 |01\rangle$ with $\tan \theta_1 = 2g(\Delta_0^2 + 4g^2)^{-1/2}$, where $\Delta_0$ is the initial qubit-resonator detuning and $g$ is the qubit-resonator field coupling. Note that for large $\Delta_0$, the eigenstate $|10\rangle \approx |10\rangle$ (in $|nm\rangle$, $n$ represents qubit state and $m$ represents the Fock state). We also present the evolution of the probability distributions of the measurement result $x_\phi$ at different times, and the waveform used for qubit frequency and for the microwave pulse. In the results presented up to page 14, we assumed a two-level model of the qubit.

We compute the Wigner function using the formula

$$W(\alpha) = Tr \left[D(-\alpha)\rho D(\alpha)e^{i\alpha a^+ a}\right],$$

where $D(\alpha) = e^{-|\alpha|^2} e^{\alpha a^+} e^{-\alpha a}$ is the usual displacement operator, $\rho$ is the density operator for the resonator field, and $a^+$ and $a$ are creation and annihilation operators for the resonator field.

The qubit frequency and the microwave pulse used in all the results shown in the following slides have the form

$$\omega_q(t) = \omega_0 + \frac{(\Delta_0 - \Delta)}{2} \left[ \text{Erf} \left( \frac{t - t_q}{\sqrt{2} \sigma_q} \right) - \text{Erf} \left( \frac{t - t_{qe}}{\sqrt{2} \sigma_{qe}} \right) \right]$$

$$B(t) = \frac{B_0}{2} \left[ \text{Erf} \left( \frac{t - t_B}{\sqrt{2} \sigma_B} \right) - \text{Erf} \left( \frac{t - t_B - \tau_B}{\sqrt{2} \sigma_B} \right) \right]$$

The parameters used for the plots presented in all the slides are:

$$\frac{\omega_0}{2\pi} = 6 \text{GHz}, \frac{\Delta_0}{2\pi} = 1 \text{GHz}, \frac{\Delta}{2\pi} = 80 \text{MHz}, \frac{g}{2\pi} = 30 \text{MHz}$$

$$\sigma_q = 4 \text{ns}, \sigma_{qe} = 1 \text{ns}, \ t_q = 10 \text{ns}, \ t_{qe} = 160 \text{ns}$$

$$\frac{B_0}{2\pi} = 0.4974 \text{GHz} (\sim 9 \text{ photons}), \sigma_B = 1 \text{ns}, \ t_B = 3 \text{ns}, \tau_B = 1 \text{ns}$$
$W_0$ — Wigner function for resonator state corresponding to an initial state $|00\rangle$

$W_1$ — Wigner function for resonator state corresponding to an initial state $|10\rangle$

**Initial state: $|00\rangle$ (qubit in state $|0\rangle$)**

$P_{0L}$ — probability for qubit to be in state $|0\rangle$

$P_{0R}$ — probability for qubit to be in state $|1\rangle$

**Initial state: $|10\rangle$ (qubit in state $|1\rangle$)**

$P_{1R}$ — probability for qubit to be in state $|1\rangle$

$P_{1L}$ — probability for qubit to be in state $|0\rangle$

$P_{0L} = 0.5117$

$P_{0R} = 0.4883$

$P_{1R} = 0.6980$

$P_{1L} = 0.3020$
\[ P_0 L = 0.8732 \]
\[ P_0 R = 0.1268 \]
\[ P_1 R = 0.9707 \]
\[ P_1 L = 0.0293 \]
\[ P_0 = 0.9988 \]
\[ P_0R = 0.0012 \]

\[ P_1 = 0.9997 \]
\[ P_1R = 0.0003 \]

\[ t = 30 \text{ ns} \]
\[ P_0^L = 1.0000 \]
\[ P_0^R = 0.0000 \]
\[ P_1^L = 0.0000 \]
\[ P_1^R = 1.0000 \]

\( \omega_0/2\pi = 6.0 \) GHz

\( \omega_0/2\pi = 6.2 \) GHz

\( \omega_0/2\pi = 6.4 \) GHz

\( \omega_0/2\pi = 6.6 \) GHz

\( \omega_0/2\pi = 6.8 \) GHz

\( \omega_0/2\pi = 7.0 \) GHz

\( \omega_0/2\pi = 7.2 \) GHz

\( \omega_0/2\pi = 7.4 \) GHz

\( \omega_0/2\pi = 7.6 \) GHz
\[ P_0 L = 0.9999 \]
\[ P_0 R = 0.0001 \]
\[ P_1 L = 0.0000 \]
\[ P_1 R = 1.0000 \]

\[ t = 60 \text{ ns} \]
\[ P_0 L = 0.9991 \]
\[ P_0 R = 0.0009 \]
\[ P_1 L = 0.0003 \]
\[ P_1 R = 0.9998 \]

\[ t = 70 \text{ ns} \]
\[ P_0 L = 0.9940 \]
\[ P_0 R = 0.0061 \]
\[ P_1 L = 0.0022 \]
\[ P_1 R = 0.9978 \]

At \( t = 80 \text{ ns} \):

- \( P_0 = 0.20 \)
- \( \omega_0 / 2\pi = 6.0 \text{ GHz} \)
- \( P_1 = 0.15 \)

Graph showing the evolution of the system with time and frequency.
\[ P_0 L = 0.9708 \]
\[ P_0 R = 0.0293 \]
\[ P_1 R = 0.9857 \]
\[ P_1 L = 0.0143 \]

\[ t = 90 \text{ ns} \]
\( P_0 L = 0.8974 \)
\( P_0 R = 0.1027 \)
\( P_1 R = 0.9394 \)
\( P_1 L = 0.0607 \)
\[ P_0 L = 0.2459 \]
\[ P_0 R = 0.7542 \]
\[ P_1 R = 0.8094 \]
\[ P_1 L = 0.1907 \]
\( P_0 L = 0.8237 \)
\( P_0 R = 0.1764 \)
\( P_1 R = 0.9167 \)
\( P_1 L = 0.0834 \)

\( t = 140 \text{ ns} \)
Effect of level $|2\rangle$ on the evolution of the Wigner function of the resonator field

In the previous slides, we assumed the two-level model of the qubit. Real superconducting qubits, however, are only slightly anharmonic oscillators, thus the effect of the next excited level $|2\rangle$ is often important as shown the following slides.

The qubit anharmonicity is given by $\mathcal{A} = \omega_q - \omega_{q,12}$. For phase and transmon superconducting qubits the anharmonicity is about $3\%$. Here we assume $\mathcal{A}/2\pi = 200$ MHz in all results shown in the following slides. All other parameters are the same as in page 2.

![Three-level model of the qubit](image)
$W_0$— Wigner function for resonator state corresponding to an initial state $|00\rangle$

$W_1$— Wigner function for resonator state corresponding to an initial state $|10\rangle$